Appendix D

Supplemental Exercises

Chapter 1 – Supplemental Exercises

Lesson 1.3

1. Find the derivative of each constant function.
   (a) $y = b$
   (b) $y = 4$
   (c) $y = -3.1$
2. Find the derivative of each linear function.
   (a) $y = mx + b$
   (b) $y = x - 4$
   (c) $y = -2x$
3. Let $T(t) = -2.1t + 63$ be the temperature in degrees Fahrenheit t hours after noon for $0 \leq t \leq 6$.
   (a) Find the temperature at noon.
   (b) Find the temperature at 3:00 p.m.
   (c) How fast was the temperature changing at 3:00 p.m.?

Lesson 1.4

4. Find the family of antiderivatives of each constant function.
   (a) $y = m$
   (b) $y = -9$
   (c) $y = 0.23$
5. Use the Fundamental Theorem of Calculus to evaluate each definite integral.
   (a) $\int_a^b m\,dx$
   (b) $\int_2^5 -6\,dx$
   (c) $\int_{-1/2}^1 4.2\,dx$
6. Stan saves money by stashing $50 per week under his mattress.
   (a) Write a definite integral that represents Stan's total accumulation of savings between the second and tenth weeks.
   (b) Use the Fundamental Theorem of Calculus to evaluate the integral from part (a), and interpret your answer in context.

Lesson 1.5

7. An ant is walking along a wire with velocity $v(t) = 2$ in/s on the time interval [0, 2], $v(t) = -t + 4$ in/s on the time interval [2, 5], and $v(t) = -1$ in/s on the time interval [5, 7].
   (a) Sketch the graph of the ant's velocity on the interval [0, 7].
   (b) Use geometry to find the ant's displacement on the interval [0, 7].
   (c) Use geometry to find the total distance traveled on the interval [0, 7].

Chapter 2 – Supplemental Exercises

Lesson 2.1

1. (a) Carefully sketch the graph of $f(x) = (x - 3)^2$ in the interval [0, 4] on the x-axis, and in the interval [0, 9] on the y-axis. Then sketch the tangent line to the graph of f at $x = 1$.
   (b) Estimate the slope of the tangent line using "rise-over-run."
   (c) Set up a table like Example 2.1.1. Use it to estimate the slope of the tangent line at $x = 1$.
   (d) Expand $(x - 3)^2$ by multiplying $(x - 3)(x - 3)$.
   (e) Use the formula for the derivative of a quadratic to find a formula for $f'(x)$.
   (f) Use the formula from part (f) to find $f'(1)$. Were your estimates good ones?

Lesson 2.3

2. (a) Write down the limit definition of the derivative of the function f.
   (b) Use the limit definition of the derivative to find the derivative of $f(x) = 3x^2 - 2x$.
3. Find the derivative of each quadratic function.
   (a) $y = ax^2 + bx + c$
   (b) $y = 0.5x^2 - 10x$
   (c) $y = (2 - x)^2$

Lesson 2.4

4. Find the derivative of each cubic function.
   (a) $y = ax^3 + bx^2 + cx + d$
   (b) $y = 1 - x + x^2 - x^3$
   (c) $y = (x + 2)(x^2 - 2x + 4)$

Lesson 2.5

5. (a) Find the equation of the tangent line to the graph of $f(x) = 2x^3 - 5x^2$ at $x = 2$.
   (b) Use the tangent line to estimate $f(2.1)$. How close are you to the actual value?
6. A beaker with radius r inches is filled with acid to a height of 4 inches. The radius of the beaker is measured to be 2 inches with a possible error in measurement of ± 0.08 inches. Estimate the propagated and relative errors in the calculated volume of acid in the beaker. (Hint: The volume of a right circular cylinder of height 4 is $V = 4\pi r^2$.)

Lesson 2.6

7. Evaluate each integral.
   (a) $\int_1^3 (-2t + 1)\, dt$
   (b) $\int_{-1}^0 (3.2u - 2.1)\, du$
   (c) $\int \left(w^2 - 4w + 10\right) dw$

Chapter 3 – Supplemental Exercises

Lesson 3.1

1. Find the derivative of each power function.
   (a) $y = 2x^{11}$
   (b) $y = \frac{7}{5x^2}$
   (c) $y = \frac{1}{x\sqrt[3]{x}}$

Lesson 3.2

2. Let $f(x) = \frac{1}{4}x^4 - 2x^2$.
   (a) Find the x-intercepts of f.
   (b) Find the critical points, the intervals of increase/decrease, and the local extrema of f.
   (c) Find the intervals of concavity and the inflection points of f.
   (d) Determine the end behavior of f.

Lesson 3.3

3. Find the derivative of each composite function.
   (a) $y = (3x - 4)^5$
   (b) $y = \sqrt[3]{x^2 - 2x}$
   (c) $y = \frac{6}{(x^2 - 2x + 1)^4}$
   (d) $y = \sqrt{9x^5 - 10}$ (use the square-root rule)
   (e) $y = \frac{1}{x^9 - 7x^3 + 1}$ (use the reciprocal rule)

Lesson 3.4

4. Find the derivative of each product function.
   (a) $y = \left(x^5 - 4\right)\left(4x^3 - 10x^2\right)$
   (b) $y = x^5\sqrt{1 + x^3}$

Lesson 3.6

5. Evaluate each integral.
   (a) $\int \left(x\sqrt{x} + \sqrt[3]{x}\right)dx$
   (b) $\int_0^1 \left(3x^9 - 4x^5 + 2\right)dx$

6. Let $f(x) = \frac{6}{x^2} - 3x^2$. Find an antiderivative F of f that passes through the point (2, –5).

Chapter 4 – Supplemental Exercises

Lesson 4.1

1. Use the quotient rule to find the derivative of each function.

   (a) $y = \frac{x-3}{x+2}$

   (b) $y = \frac{x^2+4}{\sqrt{2x+1}}$

   (c) $y = \frac{3x^3-2x}{4x^2+7}$

Lesson 4.2

2. Evaluate the following limits.

   (a) $\lim\limits_{x\to+\infty} \frac{3x^4-10x}{2x^5+4x^2+7}$

   (b) $\lim\limits_{x\to-\infty} \frac{1-7x+4x^3}{2x+10x^3}$

   (c) $\lim\limits_{x\to+\infty} \frac{9+3x^2-7x^5}{1-2x+8x^4}$

   (d) $\lim\limits_{x\to-\infty} \frac{2x^3+9x-8}{5x-2}$

   (e) $\lim\limits_{x\to-\infty} \frac{3x}{\sqrt{x^2-2x+1}}$

3. Evaluate the following limits.

   (a) $\lim\limits_{x\to 2} \frac{x^2+4x-7}{x-3}$

   (b) $\lim\limits_{x\to 0} \frac{x^3+1}{x^2-2x}$

   (c) $\lim\limits_{x\to -1} \frac{2x^5+3x+2}{x^3+2x^2+x}$

Lesson 4.3

4. Evaluate the following limits. Use L’Hôpital’s Rule if necessary.

   (a) $\lim\limits_{x\to 0} \frac{x^2-2x+3}{x^3+1}$

   (b) $\lim\limits_{x\to 3} \frac{x^3-5x^2+6x}{2x^2-5x-3}$

   (c) $\lim\limits_{x\to -2} \frac{\sqrt[3]{x^2-4}}{4x^3-10x}$

   (d) $\lim\limits_{x\to +\infty} \frac{1-\sqrt{x}}{\sqrt{x-2}}$

Chapter 5 – Supplemental Exercises

Lesson 5.2

1. Find the derivative of each function.

   (a) $y = e^x$

   (b) $y = x^2e^{-x}$

   (c) $y = \frac{5e^{t^2}}{1-3t^2}$

2. Evaluate each integral.

   (a) $\int e^x dx$

   (b) $\int e^{kx} dx$

   (c) $\int 10e^{-5t} dt$

Lesson 5.3

3. Use implicit differentiation to find $y' = \frac{dy}{dx}$.

   (a) $x^3y + 4x = 2y^2$

   (b) $\frac{1}{x} + \frac{1}{y} = 1$

   (c) $xe^y + 5y^3 = x$

Lesson 5.5

4. Find the derivative of each function.

   (a) $y = \ln x$

   (b) $y = \log_b x$

   (c) $y = \ln\left(3x^2 + 4x\right)$

   (d) $y = t \ln t$

   (e) $y = (\log_{10} x)^3$

   (f) $y = b^x$

   (g) $y = 1000(1.03)^t$

Lesson 5.6

5. Evaluate each definite integral.

   (a) $\int_0^1 1.2e^{2u} du$

   (b) $\int_2^4 \left(\frac{10}{t} - \frac{3}{t^2}\right) dt$

Chapter 6 – Supplemental Exercises

Lesson 6.2

1. Find the derivative of each function.
   (a) $y = e^{\cos x}$
   (b) $y = 16.5\sin\left(3\pi t - 18\pi\right) + 5$
2. Evaluate each integral.
   (a) $\int 5\sin(\pi x)\,dx$
   (b) $\int_{-1}^{1} \left(\cos(2t) + 1\right) dt$

Lesson 6.3

3. Find the derivative of each function.
   (a) $y = \ln(\tan x)$
   (b) $y = \sec^3(3\theta)$
   (c) $y = t^2 \tan\left(\sqrt{t}\right)$
4. Evaluate each integral.
   (a) $\int \sec^2(4t+1)\,dt$
   (b) $\int_0^{\pi/4} \sec x \tan x\,dx$
5. Evaluate each limit.
   (a) $\lim\limits_{x\to 0} \frac{\sin(10x)}{\tan(5x)}$
   (b) $\lim\limits_{x\to \pi} \frac{\cos(3x) + \left(x/\pi\right)}{\sin(2x)}$

Lesson 6.4

6. Find the derivative of each function.
   (a) $y = \arctan(\ln x)$
   (b) $y = x\sin^{-1} x$
7. Evaluate each integral.
   (a) $\int \frac{1}{\sqrt{1-(2x)^2}}\,dx$
   (b) $\int \frac{2}{9+x^2}\,dx$
   (c) $\int \frac{-1}{\sqrt{4-x^2}}\,dx$

Chapter 7 – Supplemental Exercises

Lesson 7.1

1. A spherical balloon is inflated so that the volume is increasing by 8 in^3/s. How fast is the radius of the balloon changing when the volume is 512 in^3?
2. A boat is pulled toward a dock by a rope attached to a pulley on the dock. The dock is 12 feet above the point on the bow of the boat at which the rope is attached. If the rope is pulled through the pulley at a rate of 10 ft/min, how fast is the boat approaching the dock when 50 feet of rope remain?
3. A man 6 ft tall is walking at a rate of 2 ft/s toward a stationary spotlight 20 ft high. How fast is his shadow length changing? How fast is the tip of his shadow moving?

Lesson 7.2

4. Determine each of the following for the function $y = \frac{2x^2-8}{x^2-16}$.

   (a) Domain, x-intercepts, y-intercept
   (b) Vertical asymptotes and nearby behavior
   (c) Horizontal asymptotes
   (d) Critical points
   (e) Intervals of increase/decrease, and local extrema
   (f) Intervals of concavity and inflection points
   (g) Sketch of the graph

Lesson 7.4

5. A cylindrical can with no top must be constructed using 300 in^2 of material. Find the height and radius of the can having the greatest volume.
6. A fenced-in garden is to be laid out in a rectangular area and partitioned into two sections with fencing parallel to one side. The fencing for the perimeter costs $10 per foot, while the fencing for the partition costs $5 per foot. Find the largest area that can be enclosed for a cost of $500.
7. A window consisting of a rectangle topped by a semicircle is to have a perimeter of 20 feet. Find the radius of the semicircle that maximizes the area of the window.

Lesson 7.5

8. A mass attached to a vertical spring has position $y(t)$ meters after t seconds, where y satisfies $y'' = -0.25y$. Positions below equilibrium and downward motion are considered positive. Find the position function y if the initial position is –0.3 m and the initial velocity is 2 m/s.
9. Find the general solution to the differential equation $\frac{dy}{dt} = 0.125y$.
10. Find the general solution to the differential equation $5y'' - 10y = 0$.

Chapter 8 – Supplemental Exercises

Lesson 8.2

1. Evaluate $\int_1^3 \left(x^2 - 2x\right) dx$ using the limit definition of the definite integral.

2. Use the properties of the definite integral to evaluate $\int_0^2 \left|x^2 - 3x + 2\right| dx$.

Lesson 8.4

3. If possible, use Part 1 of the Fundamental Theorem to evaluate each integral.

   (a) $\int_0^2 \frac{1}{\sqrt{x}}\, dx$

   (b) $\int_0^1 \frac{1}{3x+2}\, dx$

   (c) $\int_0^{\pi} \tan x\, dx$

Lesson 8.5

4. Use Part 2 of the Fundamental Theorem to find the derivative of each function.

   (a) $y = \int_0^x e^{t^2}\, dt$

   (b) $y = \int_x^2 \frac{1+\cos t}{1-\sin t}\, dt$

   (c) $y = \int_{3x^2}^0 \arctan(\ln t)\, dt$

5. Let $F(x) = \int_0^{x^2} \cos\left(\sqrt{t}\right) dt$.

   (a) Find $F(0)$.
   (b) Find $F'(x)$
   (c) Find $F'(\pi)$

Lesson 8.6

6. Use the method of u-substitution to evaluate each integral.

   (a) $\int_0^1 3x^2 \sqrt{1 + x^3}\, dx$

   (b) $\int_1^2 \frac{x+1}{3x^2+6x}\, dx$

   (c) $\int \frac{\sec^2(\ln x)}{x}\, dx$

   (d) $\int \frac{x}{1+(2x^2+3)^2}\, dx$

   (e) $\int \sin^3 x \cos x\, dx$