Appendix C

Activity Hints and Answers

Activity 1.1 – Average Rate of Change

1. (a) Above zero on (–2.5, 3) and (3, 5). Equal to or below zero on (–3, –2.5] and at $x = 3$.

   (b) Rising on (–3, –1) and (3, 5). The average rate of change is positive.

   (c) Falling on (–1, 3). The average rate of change is negative.

   (d) A peak at $x = -1$ means that on December 31, the temperature was higher than that of surrounding days. A valley at $x = 3$ means that on January 4, the temperature was lower than that of surrounding days.

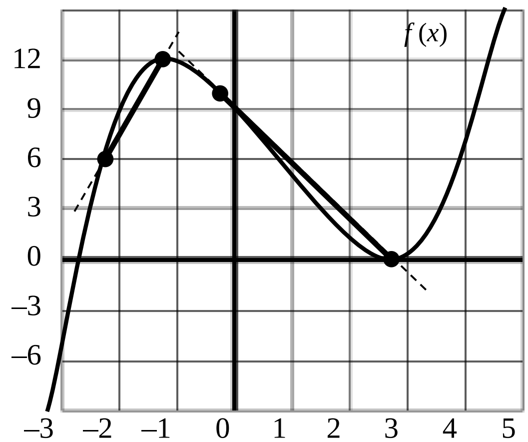
   (e) The x-intercepts are at $x = -2.5$ and at $x = 3$. These are the inputs at which the temp. was zero. The y-intercept is is at $y = 10$. This means that the temp. on January 1 was 10°C.

   (f) (i) $\dfrac{f(3)-f(0)}{3-0} = \dfrac{0-10}{3} = -10/3;$

   Between January 1 and January 4, the daily high temperature decreased by about 3.3°C per day.

   (ii) $\dfrac{f(-1)-f(-2)}{(-1)-(-2)} = \dfrac{12-6}{1} = 6;$

   Between December 30 and December 31, the daily high temperature increased by 6°C per day.



2. (a) $\dfrac{T(125)-T(25)}{125-25} = \dfrac{5.30-5.50}{125-25} = \dfrac{-0.2}{100} = -0.002$ °C per meter

   (b) $\dfrac{\Delta T}{\Delta d} = \dfrac{T(125)-T(50)}{125-50} = \dfrac{5.30-5.20}{125-50} = \dfrac{0.1}{75} \approx 0.001$ °C per meter

   (c) $\dfrac{T(200)-T(125)}{200-125} = \dfrac{6.00-5.30}{200-125} = \dfrac{0.70}{75} \approx 0.009$ °C per meter

   (d) The correct answers are (iii) and (v).

3. (a) The shape of the graph is linear.

   (b)

Interval	ΔD	Δt	$\Delta D/\Delta t$
[1, 2.5]	105	1.5	70
[0, 2]	140	2	70
[0.5, 3]	175	2.5	70

   (c) 70 mi/hr

   (d) The vehicle had a constant velocity of 70 mi/hr.

Activity 1.2 – Linear Functions

1. (a) $y - 5 = 2(x - 4)$ or $f(x) - 5 = 2(x - 4)$

   (b) $y = 2x - 3$ or $f(x) = 2x - 3$

   (c) $x = 3/2$

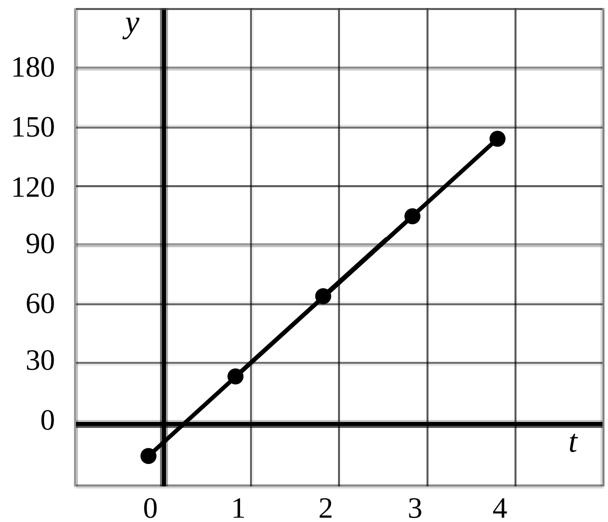
2. (a) Between 1915 and 1920, the population changed by 3100 – 3250 = –150 people, and changed at a rate of $\frac{3100-3250}{1920-1915} = -30$ people per year. The negative answers represent a decrease in population.

   (b) $P(t) = -30t + 3250$ people, where t is years after 1915.

   (c) $P(10) = -30(10) + 3250 = 2950$ people at the end of 1925.

3. (a)

Time t	Position s
0	–15
1	25
2	65
3	105
4	145



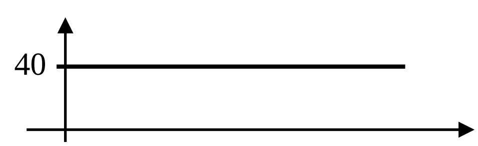
   (b) $y = s(t) = 40t - 15$ miles from Bill's house.

   (c) Set $40t - 15 = 0$ to get $40t = 15$, or $t = 15/40 = 0.375$. This is the time at which the position from Bill's house is zero. That is, they pass Bill's house after 0.375 hours.

   (d) Since $s(0) = -15$, we can conclude that the initial position was 15 miles west of Bill's.

   (e) $s'(t) = 40$ miles per hour (eastward)

4. (a) $y = s(t) = 40t + C$ miles from Bill's house



   (b) $s'(t) = 40$ miles per hour (eastward)

   (c) Infinitely many, since any line of the form $40t + C$ has a slope of 40. Examples include $40t - 10$, $4t$, and $4t + 3$. The differences between these lines are their y-intercepts.

   (d) Since the distance traveled at the start of the trip is zero, the constant $C = 0$. Therefore, $s(t) = 40t$ miles traveled.

Activity 1.3 – Derivatives of Linear Functions

1. (a) $f'(x) = 0$

   (b) $g'(x) = 0$

   (c) $h'(x) = 0$

   (d) $F'(x) = m$

   (e) $G'(x) = 9$

   (f) $H'(x) = -1$

2. (a) $v(t) = s'(t) = -2$ ft/s

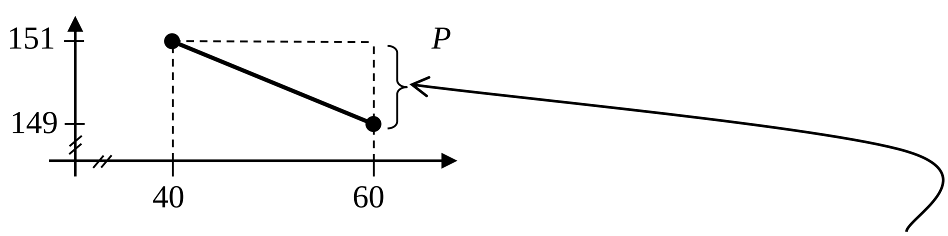
   (b) $v(10) = -2$ ft/s

   (c) $a(t) = v'(t) = 0$ ft/s^2

3. (a) The given point is (1, 22) and the slope is –0.4. Therefore, $H - 22 = -0.4(t-1)$, so $H(t) = -0.4t + 22.4$ ft^3.

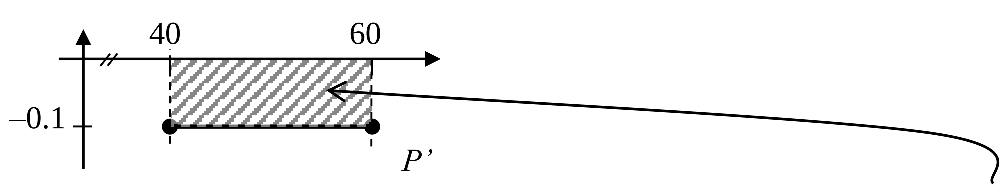
   (b) $H(5) = -0.4(5) + 22.4 = 20.4$ ft^3

4. (a) The given point is (50, 150) and the slope –0.1. Therefore, $P - 150 = -0.1(x - 50)$, so $P(x) = -0.1x + 155$ dollars, where $40 \le x \le 60$ is the number of shirts sold.

   (b)


   (c) The net change in P is $P(60) - P(40)$ = (149 dollars) – (151 dollars) = –2 dollars. The negative shows a decrease in profit.

   (d) $P'(x) = -0.1$ dollars per shirt.

   (e)


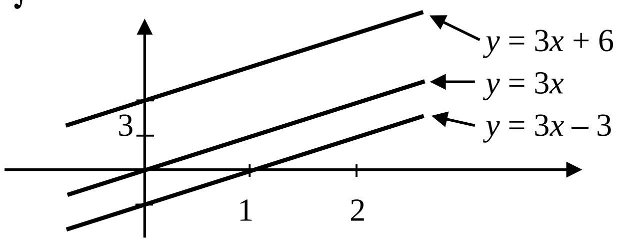
   (f) Net area bounded by P' = length × height = (20 shirts) × (–0.1 dollars/shirt) = –2 dollars

   (g) The answers are the same! This result is called the Fundamental Theorem of Calculus…

Activity 1.4 – Integrals of Constant Functions

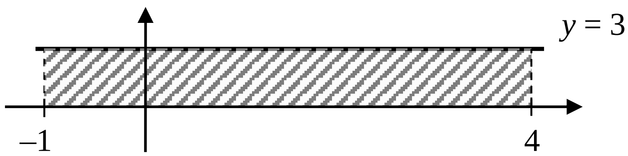
1. (a) $\int 3\, dx = 3x + C$

   (b)



   (c) $\int_{-1}^{4} 3\, dx = 3x\Big|_{-1}^{4} = 3(4) - 3(-1) = 15$

   (d)



2. (a) $\int 1\, dx = x + C$

   (b) $\int 5.4\, dt = 5.4t + C$

   (c) $\int -62\, du = -62u + C$

   (d) $\int_{2}^{5} -1\, dx = (-x)\Big|_{2}^{5} = (-5) - (-2) = -3$

   (e) $\int_{0}^{3} 4.32\, dx = (4.32x)\Big|_{0}^{3} = 4.32(3) - 4.32(0) = 12.96$

   (f) $\int_{-3}^{2} -0.02\, dv = (-0.02v)\Big|_{-3}^{2} = (-0.02(2)) - (-0.02(-3)) = -0.1$

3. (a) $\int v(t)\, dt = \int -45\, dt = -45t + C$ miles from Bill's at t hours

   (b) $\int_{0}^{2} v(t)\, dt = \int_{0}^{2} -45\, dt = (-45t)\Big|_{0}^{2} = \big(-45(2)\big) - \big(-45(0)\big) = -90$

   Over the first 2 hours, Bill and Sally traveled a net distance of 90 miles westward.

   (c) No, we need to know the distance from Bill's at the start of the trip.

   (d) Since $s(t) = -45t + C$ and $s(0) = 200$, $s(0) = C = 200$. Therefore, the distance function is $s(t) = -45t + 200$ and $s(2) = -45 \cdot 2 + 200 = 110$ miles east of Bill's house.

   (e) Part (b) is the net distance traveled, but Part (d) is the distance from Bill's.

4. $\int_{-1}^{0} 2\, dx = (2x)\Big|_{-1}^{0} = 2(0) - 2(-1) = 2$

5. $\int_{-1}^{1} (2x - 8)\, dx =$ – [area of trapezoid, or area of rectangle and triangle] = –16

6. If $f'(x) = 2$, then $f(x) = 2x + C$ and $f(1) = 2 + C = 7$. Hence, $C = 5$ and $f(x) = 2x + 5$.

Activity 1.5 – Rectilinear Motion

1. Position $s(t) = 35 - t$ feet
   Velocity $v(t) = -1$ ft/s
   Speed $|v(t)| = 1$ ft/s
   Acceleration $a(t) = 0$ ft/s^2

   Position $s(5) = 30$ ft
   Velocity $v(5) = -1$ ft/s
   Speed $|v(5)| = 1$ ft/s
   Acceleration $a(5) = 0$ ft/s^2

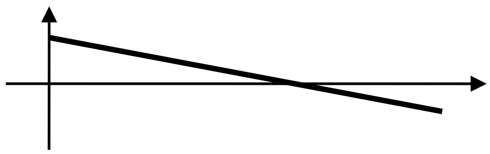
2. (a) $s(t) = -20t + 100$ m

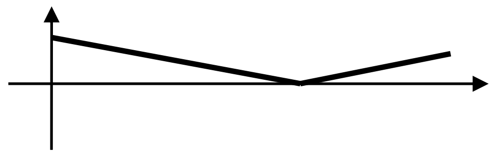
   (b) Setting $-20t + 100 = 0$ yields $t = 5$ s.

   (c) $\int_{9.5}^{12.5} -20\,dt = (-20t)\Big|_{9.5}^{12.5} = (-20(12.5)) - (-20(9.5)) = -60\text{ m}$

   (d) $\int_{9.5}^{12.5} 20\,dt = (20t)\Big|_{9.5}^{12.5} = 20(12.5) - 20(9.5) = 60\text{ m}$

3. (a)





   (b) Setting $10 - 4t = 0$ yields $t = 10/4 = 2.5$ s.

   (c) $\int_0^4 (10 - 4t)\,dt = \frac{1}{2}(2.5)(10) - \frac{1}{2}(1.5)(6) = 8\text{ m}$

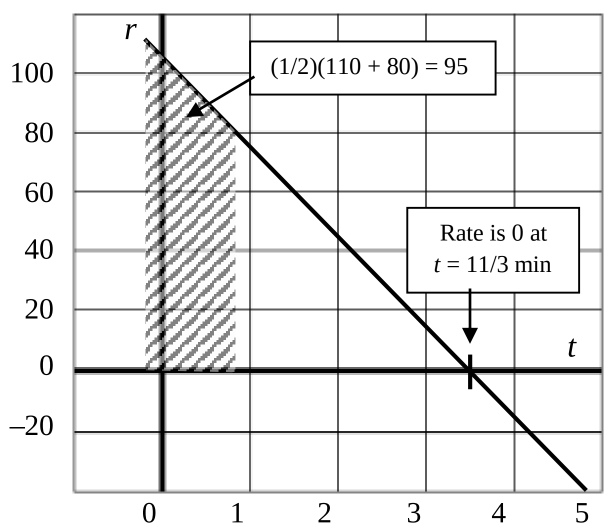
   (d) $\int_0^4 |10 - 4t|\,dt = \frac{1}{2}(2.5)(10) + \frac{1}{2}(1.5)(6) = 17\text{ m}$

4. (a) Linear because for each unit change in t, there is a constant change of -30 in r. The slope is -30 and the initial value (y-int.) is 110. Therefore, $r(t) = -30t + 110$ gal/min.

   (b) Setting $-30t + 110 = 0$ yields $t = 11/3$ min.

   (c)

t (min)	$r(t)$ (gal/min)	$V(t)$ (gal)
0	110	500
1	80	595
2	50	660
3	20	695
4	–10	700
5	–40	675



   (e) Quadratic

   (f) $V(t) = -15t^2 + 110t + 500$ gallons, where t is minutes after the malfunction

Activity 2.1 – Derivatives of Quadratic Functions

1. (a) 19.4 ft/s

   (b) 20.84 ft/s

   (c) 20.984 ft/s

   (d) 21 ft/s

   (e) $v(t) = 85 - 32t$; $v(2) = 21$ ft/s.

   (f) $a(t) = -32$ ft/s^2

2. (a) $f'(x) = 4x - 10$; $f''(x) = 4$

   (b) $y'(w) = -1 + w$; $y''(w) = 1$

   (c) $h'(r) = 32r - 24$; $h''(r) = 32$

3. $P'(t) = 8t + 34$ bacteria per hour; $P'(2) = 50$ bacteria per hour

4. (a) $f(4) = 5$

   (b) $f'(4) = -2$

   (c) $y - 5 = -2(x - 4)$

Activity 2.2 – Analyzing Quadratic Functions

1. (a) $x = -4, 4$

   (b) $x = -1, 0, 1$

   (c) $x = 0$ (Factor as $x^2\left(\frac{1}{2}x^2 + 2\right) = 0$, and note that $\frac{1}{2}x^2 + 2$ is never zero.)

   (d) $x = 4/5, 1$

   (e) $x = -3, -1, 1, 3$

2. (a) 5°F

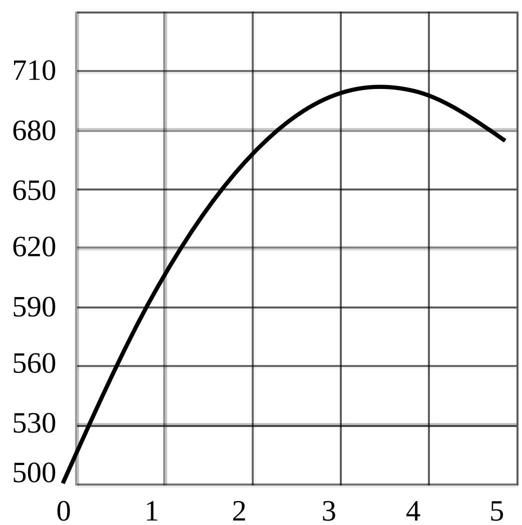
   (b) 1:00 a.m. and 2:30 a.m.

   (c) $T'(t) = 4t - 7$; rising by 5°F per hour

   (d) The low temperature occurred at 1:45 a.m. and was approximately –6°F.

   (e) Falling on (0, 7/4); rising on (7/4, 4)

3. (a)



   (b) If $V'(t) = -30t + 110 = 0$, then $t = 11/3$. The maximum volume is $V(11/3) = 701.67$ gal.

   (c) $V(t) = -15t^2 + 110t + 500 = 0$ when $t = \frac{-110 \pm \sqrt{110^2 - 4\cdot(-15)\cdot 500}}{2\cdot(-15)} \approx -3.17$ or 10.5. We cast out the negative solution and find that it takes 10.5 minutes for the tank to empty.

4. Since $C'(x) = 0.0004x + 7$, we have $C'(1000) = 7.4$. The marginal cost is \$7.40.

Activity 2.3 – Definition and Properties of the Derivative

1. (a) $$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0}\frac{\left(9(x+\Delta x)+2\right)-\left(9x+2\right)}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{\left(9x+9\Delta x+2\right)-\left(9x+2\right)}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{9x+9\Delta x+2-9x-2}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{9\Delta x}{\Delta x} \\ &= \lim_{\Delta x\to 0} 9 \\ &= 9 \end{aligned}$$

   (b) $f'(x)=4x$

   (c) $f'(x)=6x-2$

2. $$\begin{aligned} y' &= \lim_{\Delta x\to 0}\frac{(x+\Delta x)^3-x^3}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{x^3+3x^2\cdot\Delta x+3x\cdot\Delta x^2+\Delta x^3-x^3}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{3x^2\cdot\Delta x+3x\cdot\Delta x^2+\Delta x^3}{\Delta x} \\ &= \lim_{\Delta x\to 0}(3x^2+3x\cdot\Delta x+\Delta x^2) \\ &= 3x^2 \end{aligned}$$

3. (a) $f'(x)=\left(ax^3+bx^2+cx+d\right)'=\left(ax^3\right)'+\left(bx^2\right)'+\left(cx\right)'+\left(d\right)'=3ax^2+2bx+c$

   (b) (i) $f'(x)=6x^2-2x$; $f''(x)=12x-2$

   (ii) $g'(x)=-3.6x^2+4.6x+0.6$; $g''(x)=-7.2x+4.6$

4. $$\begin{aligned} \left(k\cdot f(x)\right)' &= \lim_{\Delta x\to 0}\frac{k\cdot f(x+\Delta x)-k\cdot f(x)}{\Delta x} \\ &= k\cdot\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} \\ &= k\cdot f'(x) \end{aligned}$$

$$\begin{aligned} \left(f(x)+g(x)\right)' &= \lim_{\Delta x\to 0}\frac{\left[f(x+\Delta x)+g(x+\Delta x)\right]-\left[f(x)+g(x)\right]}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)+g(x+\Delta x)-g(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0}\left[\frac{f(x+\Delta x)-f(x)}{\Delta x}+\frac{g(x+\Delta x)-g(x)}{\Delta x}\right] \\ &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x} \\ &= f'(x)+g'(x) \end{aligned}$$

Activity 2.4 – Analyzing Cubic Functions

1. (a) The graph is…………………….. Increasing $\boxed{\text{Decreasing}}$
   The derivative (slope) is……….. Positive $\boxed{\text{Negative}}$
   The derivative (slope) is……….. $\boxed{\text{Increasing}}$ Decreasing

   (b) The graph is…………………….. $\boxed{\text{Increasing}}$ Decreasing
   The derivative is……………….. $\boxed{\text{Positive}}$ Negative
   The derivative is……………….. $\boxed{\text{Increasing}}$ Decreasing

2. (a) The graph is…………………….. $\boxed{\text{Increasing}}$ Decreasing
   The derivative is……………….. $\boxed{\text{Positive}}$ Negative
   The derivative is……………….. Increasing $\boxed{\text{Decreasing}}$

   (b) The graph is…………………….. Increasing $\boxed{\text{Decreasing}}$
   The derivative is……………….. Positive $\boxed{\text{Negative}}$
   The derivative is……………….. Increasing $\boxed{\text{Decreasing}}$

3. $y' = 3x^2 - 4x - 5$; $y'' = 6x - 4 = 0$ yields $x = 2/3$. A sign test shows that y is concave up on $(2/3, \infty)$ and concave down on $(-\infty, 2/3)$. The inflection point is at $x = 2/3$ and the coordinates are $(2/3, 56/27)$.

4. (a) $s'(t) = 3t^2 - 18t + 27 = 0$ yields $t = 3$.

   s': $+$ | $+$ (at 3)

   $s''(t) = 6t - 18 = 0$ yields $t = 3$.

   s'': $-$ | $+$ (at 3)

   (b) Speeding up on $(3, \infty)$; slowing down on $(-\infty, 3)$.

5. (a) $(x - 5)(x^2 + 5x + 25)$; x-intercept at $x = 5$.

   (b) $(x + 4)(x^2 - 4x + 16)$; x-intercept at $x = -4$.

6. $x^3 + 2x^2 - 5x - 6 = (x + 1)(x^2 + x - 6) = (x + 1)(x + 3)(x - 2)$; the solutions are $x = -1, -3, 2$.

7. $x^3 - 5x^2 - 12x + 60 = x^2(x-5) - 12(x-5) = (x-5)(x^2 - 12) = (x-5)\left(x + \sqrt{12}\right)\left(x - \sqrt{12}\right)$; the solutions are $x = 5$, $\sqrt{12}$, $-\sqrt{12}$.

Activity 2.5 – Linear Approximation

1. (a) $\frac{dy}{dx} = -6x^2 + 2x$

$$\frac{d^2y}{dx^2} = -12x + 2$$

(b) $\left.\frac{dy}{dx}\right|_{x=-1} = \left(-6x^2 + 2x\right)\Big|_{x=-1} = -6(-1)^2 + 2(-1) = -8$

$$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = \left(-12x + 2\right)\Big|_{x=-1} = -12(-1) + 2 = 14$$

2. (a) $\Delta y = y(2.03) - y(2) = 2(2.03)^3 - 2(2)^3 = 0.731$

(b) $dy = 6x^2dx = 6(2)^2(.03) = 0.720$

3. (a) $f(2) = 4$; $f'(2) = 12$

(b) $y - 4 = 12(x - 2)$; $y = 12x - 20$

(c) $f(2.01) \approx 12(2.01) - 20 = 4.12$

(d) $f(2.01) = 2(2.01)^3 - 3(2.01)^2 = 4.120902$

4. (a) $dA = 10xdx = 10(36)(\pm 0.125) = \pm 45$ in^2

(b) $\frac{dA}{A} = \frac{10xdx}{5x^2} = \frac{10(36)(\pm 0.125)}{5(36)^2} \approx \pm 0.0069 = \pm 0.69\%$

5. $dV = 4\pi r^2 dr = 4\pi(19)^2(\pm 0.5) \approx \pm 2268.23$ cm^3

$$\frac{dV}{V} = \frac{4\pi r^2 dr}{(4/3)\pi r^3} = \frac{4\pi(19)^2(\pm 0.5)}{(4/3)\pi(19)^3} \approx \pm 0.0789 = \pm 7.89\%$$

6. $dA = 2\pi r dr = 2\pi(24)(\pm 0.1) \approx \pm 15.08$ cm^2

$$\frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = \frac{2\pi(24)(\pm 0.1)}{\pi(24)^2} \approx \pm 0.0083 = \pm 0.83\%$$

7. $dV = 74\pi r dr = 74\pi(12)(\pm 0.15) \approx \pm 418.46$ m^3

$$\frac{dV}{V} = \frac{74\pi r dr}{37\pi r^2} = \frac{74\pi(12)(\pm 0.15)}{37\pi(12)^2} = \pm 0.025 = \pm 2.5\%$$

Activity 2.6 – Integrals of Linear and Quadratic Functions

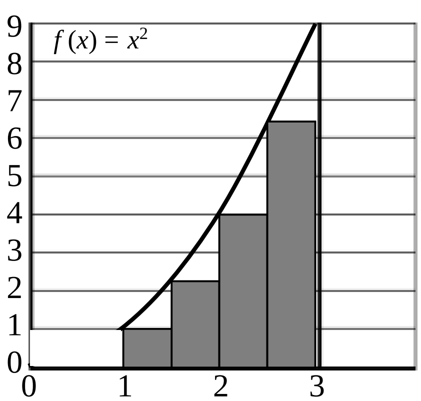
1. (a) $5x^2 + C$
   (b) $\frac{3}{2}t^2 + 4t + C$
   (c) $u - \frac{1}{2}u^2 + C$

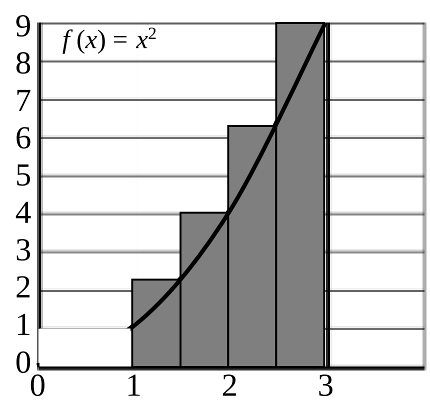
2. (a) $\left(5x^2\right)\Big|_1^2 = 5(2)^2 - 5(1)^2 = 15$
   (b) $\left(\frac{3}{2}t^2 + 4t\right)\Big|_0^1 = \left(\frac{3}{2}(1)^2 + 4(1)\right) - \left(\frac{3}{2}(0)^2 + 4(0)\right) = \frac{11}{2}$
   (c) $\left(u - \frac{1}{2}u^2\right)\Big|_{-2}^3 = \left((3) - \frac{1}{2}(3)^2\right) - \left((-2) - \frac{1}{2}(-2)^2\right) = \frac{5}{2}$

3. (a) $\int\left(2x^2 - x + 7\right)dx = \frac{2}{3}x^3 - \frac{1}{2}x^2 + 7x + C$
   (b) $\int\left(32 - 6w + w^2\right)dw = 32w - 3w^2 + \frac{1}{3}w^3 + C$

4. $\int_5^8 (10 + 8t)dt = \left(10t + 4t^2\right)\Big|_5^8 = \left(10(8) + 4(8)^2\right) - \left(10(5) + 4(5)^2\right) = 186$ cows

5. (a) $v(t) = -32t + 16$ ft/s
   (b) $s(t) = -16t^2 + 16t + 175$ ft
   (c) Set $-16t^2 + 16t + 175 = 0$ and solve for t (using the quadratic formula) to get t = 3.84 s.
   (d) $v(3.84) \approx -107$ ft/s

6. 

7. 

8. $L_4 = \left(1^2 + 1.5^2 + 2^2 + 2.5^2\right)\cdot 0.5 = 6.75$; $R_4 = \left(1.5^2 + 2^2 + 2.5^2 + 3^2\right)\cdot 0.5 = 10.75$.
   That is, $6.75 \le \int_1^3 x^2 dx \le 10.75$.

9. Δx = (3 – 1)/8 = 0.25.

Activity 3.1 – Power Functions

1. (a) $y' = 0$

   (b) $y' = -1$

   (c) $y' = \frac{1}{3}$

   (d) $y' = x$

   (e) $\frac{dy}{dx} = -\frac{6}{7x^4}$

   (f) $\frac{dy}{dx} = 30x^9$

   (g) $\frac{dy}{dx} = -\frac{8}{5\sqrt[5]{x}}$

   (h) $\frac{dy}{dx} = -\frac{8}{3\sqrt[3]{x^5}}$

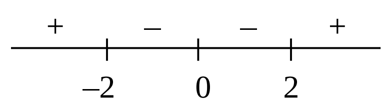
2. (a) $y' = \frac{1}{2\sqrt{x}}$

   (b) $y' = -\frac{1}{x^2}$

3. (a) $f'(r) = -\frac{8}{3r^3} - \frac{1}{2r^2}$; $f'(2) = -\frac{8}{3(2)^3} - \frac{1}{2(2)^2} = -\frac{11}{24}$

   (b) $g'(t) = \frac{25}{2}t^{3/2} - \frac{1}{3t^{2/3}}$; $g'(1) = \frac{25}{2} - \frac{1}{3} = \frac{73}{6}$

4. (a) Set $f'(x) = 3 - \frac{12}{x^2} = 0$ to get $x = 2, -2$.



   (b) local minimum at (2, 12)

   (c) local maximum at (–2, –12)

5. (a) The domain of $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ is $x \neq 0$, so $y = \sqrt{x}$ is not differentiable at $x = 0$.

   (b) The domain of $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$ is $x > 0$, so $y = \sqrt{x}$ is not differentiable for $x \leq 0$.

6. (a)
$$\frac{d}{dx}\left(\sqrt{x}\right) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

   (b)
$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{\Delta x \to 0} \frac{\left(\frac{1}{x+\Delta x} - \frac{1}{x}\right)}{\Delta x} \cdot \frac{x(x+\Delta x)}{x(x+\Delta x)}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x \cdot x(x+\Delta x)}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$$
$$= -\frac{1}{x^2}$$

Activity 3.2 – Polynomial Functions

1. (a) $f'(x) = -8x^7 + 10x^4 - 36x + 1$

   (b) $f''(x) = -56x^6 + 40x^3 - 36$

   (c) $f'''(x) = -336x^5 + 120x^2$

2. (a) $v(t) = 3t^2 - 20$ m/s

   (b) $a(t) = 6t$ m/s^2

   (c) $v(2) = -8$ m/s; $a(2) = 12$ m/s^2

   (d) Moving left ($v < 0$); slowing down (v and a have opposite signs)

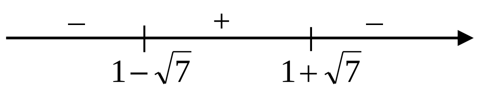
3. (a) $\lim_{x\to+\infty}\left(3x^2 - 12x^6\right) = -\infty$; $\lim_{x\to-\infty}\left(3x^2 - 12x^6\right) = -\infty$

   (b) $\lim_{x\to+\infty}\left(-13x^5 + 5x^2\right) = -\infty$; $\lim_{x\to-\infty}\left(-13x^5 + 5x^2\right) = +\infty$

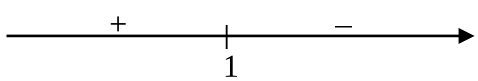
4. (a) Setting $-x^3 + 3x^2 + 18x = -x(x+3)(x-6) = 0$ yields $x = -3, 0, 6$.

   (b) Setting $f'(x) = -3x^2 + 6x + 18 = 0$ yields $x = 1 \pm \sqrt{7}$ (using the quadratic formula).

   Relative minimum at $x = 1 - \sqrt{7}$; relative maximum at $x = 1 + \sqrt{7}$.



   (c) Setting $f''(x) = -6x + 6 = 0$ yields $x = 1$. Inflection point at $x = 1$.



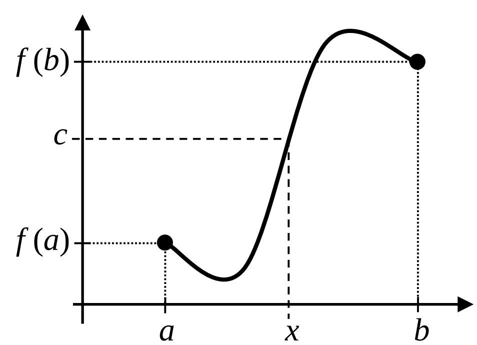
   (d) $\lim_{x\to\infty}(-x^3 + 3x^2 + 18x) = \lim_{x\to\infty}(-x^3) = -\infty$; $\lim_{x\to-\infty}(-x^3 + 3x^2 + 18x) = \lim_{x\to-\infty}(-x^3) = +\infty$

5. (b) yes;

   (c) $f(x) = c$;

   (d) continuity

   (e) <u>Intermediate Value Theorem</u>:
   If f is a <u>continuous</u> function defined on the closed interval $[a, b]$, and c is any number between <u>$f(a)$</u> and <u>$f(b)$</u>, then there exists a number x between <u>a</u> and <u>b</u> such that <u>$f(x) = c$</u>.



   (f) an x-intercept

Activity 3.3 – Composite Functions

1. (a) $\frac{dL}{dP} = 0.06$ ppm/h; (b) $\frac{dP}{dt} = 4$ h/mo; (c) $\frac{dL}{dt} = \frac{dL}{dP} \cdot \frac{dP}{dt} = (0.06)(4) = 0.24$ ppm/mo

2. (a) $y' = 10x^9$

   (b) $y' = -12x^{-7}$

   (c) $y' = 5(3x)^4 \cdot 3 = 15(3x)^4 = 1215x^4$

   (d) $y' = 8\left(x^3 - 7x^2\right)^3 \cdot \left(3x^2 - 14x\right)$

   (e) $y' = -2\left(x^4 + 6x^2 - 9\right)^{-3} \cdot \left(4x^3 + 12x\right) = \frac{-2\left(4x^3+12x\right)}{\left(x^4+6x^2-9\right)^3}$

   (f) $y' = \frac{1}{5}(2x+5)^{-4/5} \cdot 2 = \frac{2}{5(2x+5)^{4/5}}$

   (g) $y' = \frac{4}{3}\left(1 + 2x + 3x^2\right)^{1/3} \cdot (2 + 6x)$

   (h) $y' = -10(1-10x)^{-3} \cdot (-10) = \frac{100}{(1-10x)^3}$

   (i) $y' = \frac{3}{5}\left(4x^2 + 7\right)^{-6/5} \cdot 8x = \frac{24x}{5\left(4x^2+7\right)^{6/5}}$

3. (a) $\frac{dy}{dx} = \frac{1}{2\sqrt{5x}} \cdot 5 = \frac{5}{2\sqrt{5x}}$

   (b) $\frac{dy}{dx} = -\frac{5}{2\sqrt{2x^2-10x}} \cdot (4x - 10) = \frac{-10x+25}{\sqrt{2x^2-10x}}$

4. (a) $\frac{dy}{dx} = -\frac{1}{(7x+2)^2} \cdot 7 = \frac{-7}{(7x+2)^2}$

   (b) $\frac{dy}{dx} = -\frac{7}{(3x^5-1)^2} \cdot 15x^4 = -\frac{-105x^4}{(3x^5-1)^2}$

5. (a) $V'(t) = -0.2$, so the volume is decreasing by –0.2 liters per minute.

   (b) $P(t) = \frac{nRT}{V(t)} = \frac{(15)(0.08205)(215)}{6 - 0.2t} = \frac{264.61125}{6 - 0.2t}$ atmospheres

   (c) $P'(t) = -\frac{264.61125}{(6-0.2t)^2} \cdot (-0.2) = \frac{52.92225}{(6-0.2t)^2}$ atmospheres per minute

   (d) $V(7) = 6 - 0.2(7) = 4.6$ liters; $P(7) = \frac{264.61125}{6 - 0.2(7)} = 57.52418$ atmospheres

   (e) $P'(7) = \frac{52.92225}{(6-0.2(7))^2} = 2.50105$ atmospheres per minute

Activity 3.4 – Products of Functions

1. (a) $f'(x) = (6)(x^2 + 2x - 2) + (6x - 7)(2x + 2)$

   (b) $g'(x) = \left(5(2x-3)^4 \cdot 2\right)(5x^2 + 2x) + (2x-3)^5(10x + 2)$

   (c) $s'(t) = (3t^2)\left(\sqrt{8t+2}\right) + (t^3)\left(\frac{1}{2\sqrt{8t+2}} \cdot 8\right)$

2. (a) $y' = 10(x^2 - 3x + 1)^9 \cdot (2x - 3)$

   $y'' = \left(90(x^2 - 3x + 1)^8 \cdot (2x - 3)\right) \cdot (2x - 3) + \left(10(x^2 - 3x + 1)^9\right)(2)$

   (b) $y' = \frac{1}{3}(7 - 2x^2)^{-2/3} \cdot (-4x)$

   $y'' = \left(-\frac{2}{9}(7 - 2x^2)^{-5/3} \cdot (-4x)\right)(-4x) + \left(\frac{1}{3}(7 - 2x^2)^{-2/3}\right)(-4)$

3. $f'(x) = \left(\frac{5}{7}x^{-2/7}\right)(x-9)^2 + \left(x^{5/7}\right)(2(x-9))$

   $= x^{-2/7}(x-9)\left(\frac{5}{7}(x-9) + 2x\right)$

   $= \dfrac{(x-9)\left(\frac{19}{7}x - \frac{45}{7}\right)}{x^{2/7}}$

   Horizontal tangents at $x = 9$ and at $x = 45/19$. Vertical tangent at $x = 0$.

4. (a) $R'(t) = S'(t)P(t) + S(t)P'(t)$ dollars per day

   (b) $R'(60) = S'(60)P(60) + S(60)P'(60) = (-15)(199) + (1290)(2) = -405$ dollars per day;
   That is, the revenue was decreasing by $405 per day.

5. (a) $(f + \Delta f)(g + \Delta g) = (f \cdot g) + (\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)$

   (b) $\Delta(f \cdot g) = (f + \Delta f)(g + \Delta g) - (f \cdot g) = (\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)$

   (c) $\dfrac{\Delta(f \cdot g)}{\Delta x} = \dfrac{(\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)}{\Delta x} = \dfrac{\Delta f}{\Delta x} \cdot g + f \cdot \dfrac{\Delta g}{\Delta x} + \dfrac{\Delta f}{\Delta x} \cdot \Delta g$

   (d) Since $\dfrac{df}{dx}$ exists (is finite), and $\Delta g \to 0$ as $\Delta x \to 0$, we have $\lim_{\Delta x \to 0}\left(\dfrac{\Delta f}{\Delta x} \cdot \Delta g\right) = \dfrac{df}{dx} \cdot 0 = 0$.

   (e) $\dfrac{d}{dx}(f \cdot g) = \lim_{\Delta x \to 0} \dfrac{\Delta(f \cdot g)}{\Delta x} = \lim_{\Delta x \to 0}\left(\dfrac{\Delta f}{\Delta x} \cdot g + f \cdot \dfrac{\Delta g}{\Delta x} + \dfrac{\Delta f}{\Delta x} \cdot \Delta g\right) = \dfrac{df}{dx} \cdot g + f \cdot \dfrac{dg}{dx}$

Activity 3.5 – Piecewise Functions

1. (a) $g'(x)=\begin{cases}2x-6, & \text{if } x<-1\\ a, & \text{if } x>-1\end{cases}$

   (b) Set $2(-1)-6=a$ to get $a=-8$; set $(-1)^2-6(-1)+5=(-8)(-1)+b$ to get $b=4$.

2. $a = 1/2$; $b = 8$

3. (a) $f(-3)=6$ (b) $f(-1)=-2$ (c) $f(0)=1$

   (d) $f(1)=2$ (e) $f(5)=-2$

4. (a) $\lim_{x\to-1^-} f(x)=-2$ (b) $\lim_{x\to-1^+} f(x)=0$ (c) $\lim_{x\to-1} f(x)=$ DNE

   (d) $\lim_{x\to1^-} f(x)=2$ (e) $\lim_{x\to1^+} f(x)=2$ (f) $\lim_{x\to1} f(x)=2$

   (g) No (h) Yes

5. $f'(x)=\begin{cases}2x, & \text{if } x<-1\\ 1, & \text{if } -1<x<1\\ -1, & \text{if } x>1\end{cases}$

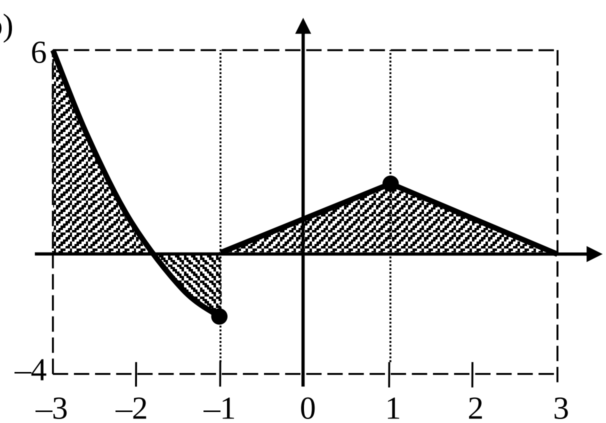
6. (a) $\lim_{x\to-1^-} f'(x)=-2$ (b) $\lim_{x\to-1^+} f'(x)=1$ (c) $\lim_{x\to-1} f'(x)=$ DNE

   (d) $\lim_{x\to1^-} f'(x)=1$ (e) $\lim_{x\to1^+} f'(x)=-1$ (f) $\lim_{x\to1} f'(x)=$ DNE

   (g) No (h) No

7. (a) The function f is not continuous on [–3, 3].

   (b)



   (c) $\int_{-3}^{3} f(x)dx=\int_{-3}^{-1}(x^2-3)dx+\int_{-1}^{1}(x+1)dx+\int_{1}^{3}(3-x)dx=\frac{8}{3}+2+2=\frac{20}{3}$

Activity 3.6 – Integrals of Polynomials

1. (a) $-\frac{1}{2}t^4+\frac{5}{3}t^3+\frac{1}{2}t^2-4t+C$

   (b) $\frac{9}{5}x^{5/3}+\frac{7}{x}+C$

   (c) $\frac{2}{3}x^{3/2}+C$

   (d) $\frac{3}{4}x^{4/3}+10x^{1/2}+C$

   (e) $\int_{-1}^{1}(3u+2)^2\,dx=\int_{-1}^{1}\left(9u^2+12u+4\right)dx=\left(3u^3+6u^2+4u\right)\Big|_{-1}^{1}=14$

   (f) $\int_1^4\left(\frac{2x^3-32}{2x^3}\right)dx=\int_1^4\left(1-\frac{16}{x^3}\right)dx=\left(x+\frac{8}{x^2}\right)\Big|_1^4=-\frac{9}{2}$

2. (a) $F(x)=\int\left(7x^{-3}-6x^{-5}\right)dx=-\frac{7}{2x^2}+\frac{3}{2x^4}+C$; set $F(1)=-\frac{7}{2}+\frac{3}{2}+C=3$ to get $C=5$;

   $F(x)=-\frac{7}{2x^2}+\frac{3}{2x^4}+5$

   (b) Set $F(2)=-\frac{7}{8}+\frac{3}{32}+C=-\frac{41}{32}$ to get $C=-\frac{1}{2}$; $F(x)=-\frac{7}{2x^2}+\frac{3}{2x^4}-\frac{1}{2}$

3. Let $f(x)=x$ and let $g(x)=x$.

   The integral of the product is $\int f(x)\cdot g(x)\,dx=\int x^2dx=\frac{1}{3}x^3$, but

   the product of the integrals is $\int f(x)\,dx\cdot\int g(x)\,dx=\int xdx\cdot\int xdx=\frac{1}{2}x^2\cdot\frac{1}{2}x^2=\frac{1}{4}x^4$.

4. (a) (i) Since $\frac{d}{dx}\left(k\cdot F(x)\right)=k\cdot f(x)$, it follows that $\int k\cdot f(x)dx=k\cdot F(x)$.

   (ii) Since $\frac{d}{dx}\left(F(x)\right)=f(x)$, it follows that $\int f(x)dx=F(x)$.

   (iii) From (ii), then (i), $k\cdot\int f(x)dx=k\cdot F(x)=\int k\cdot f(x)dx$.

   (b) (i) Since $\frac{d}{dx}\left(F(x)\pm G(x)\right)=f(x)\pm g(x)$, we have $\int(f(x)\pm g(x))dx=F(x)\pm G(x)$.

   (ii) Since $\frac{d}{dx}\left(F(x)\right)=f(x)$, we have $\int f(x)dx=F(x)$.

   (iii) Since $\frac{d}{dx}\left(G(x)\right)=g(x)$, we have $\int g(x)dx=G(x)$.

   (iv) From (ii) and (iii), then (i), $\int f(x)dx\pm\int g(x)dx=F(x)\pm G(x)=\int(f(x)\pm g(x))dx$.

Activity 4.1 – Analyzing Rational Functions

1. (a) $N(1) = (1)^3 - 5(1)^2 + 7(1) - 3 = 0$

   (b) $g(x) = x^2 - 4x + 3$

   (c) $N(x) = (x-1)^2(x-3)$

   (d) $f(x) = \dfrac{(x-1)^2(x-3)}{(x-1)(x+1)}$

   (e) $x = 1, -1$

   (f) $x = 3$

   (g) $y = 3$

   (h) $x = -1$

   (i) $x = 1$

   (j) $f(x) = (x-5) + \dfrac{8x-8}{x^2-1}$

3. (a) $y' = \dfrac{(1)(2x-3)-(x)(2)}{(2x-3)^2} = \dfrac{-3}{(2x-3)^2}$

   (b) $y' = \dfrac{(2x+3)(x+4)-(x^2+3x)(1)}{(x+4)^2} = \dfrac{x^2+8x+12}{(x+4)^2}$

   (c) $y' = \dfrac{(2x)(3x^2-9x+6)-(x^2-2)(6x-9)}{(3x^2-9x+6)^2} = \dfrac{-9x^2+24x-18}{(3x^2-9x+6)^2}$

   (d) $y' = \dfrac{\left(\frac{1}{2\sqrt{x-2}}\cdot(1)\right)(x^2-9)-\left(\sqrt{x-2}\right)(2x)}{(x^2-9)^2} = \dfrac{-3x^2+8x-9}{2\sqrt{x-2}(x^2-9)^2}$

4. Set $y' = \dfrac{x^2+8x+12}{(x+4)^2} = \dfrac{(x+2)(x+6)}{(x+4)^2} = 0$ to get $x = -2, -6$.

5. (a) $C(3) = 0.032$ mg/cm^3

   (b) $C'(t) = \dfrac{(0.16)(t^2+6)-(0.16t)(2t)}{(t^2+6)^2} = \dfrac{-0.16t^2+0.96}{(t^2+6)^2}$ mg/cm^3 per hour

   (c) $C'(3) \approx -0.002$ mg/cm^3 per hour

Activity 4.2 – Horizontal and Vertical Asymptotes

1. Since $\lim\limits_{x\to+\infty}\frac{9x^4-6x^3+2x}{2x^4+7x-1}=\lim\limits_{x\to+\infty}\frac{9x^4}{2x^4}=\lim\limits_{x\to+\infty}\frac{9}{2}=\frac{9}{2}$, the line $y=\frac{9}{2}$ is a horizontal asymptote.
   Similarly for $x\to-\infty$.

2. (a) $\lim\limits_{x\to+\infty}\frac{5x+x^2}{6-4x^2}=\lim\limits_{x\to+\infty}\frac{x^2}{-4x^2}=\lim\limits_{x\to+\infty}\frac{1}{-4}=-\frac{1}{4}$

   (b) $\lim\limits_{x\to-\infty}\frac{x^5-10x^2}{3x^6+x^4+1}=\lim\limits_{x\to-\infty}\frac{x^5}{3x^6}=\lim\limits_{x\to-\infty}\frac{1}{3x}=0$

   (c) $\lim\limits_{x\to+\infty}\frac{2x+x^2}{81-x}=\lim\limits_{x\to+\infty}\frac{x^2}{-x}=\lim\limits_{x\to+\infty}\frac{x}{-1}=-\infty$

   (d) $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x-4}=\lim\limits_{x\to-\infty}\frac{\sqrt{x^2}}{3x}=\lim\limits_{x\to-\infty}\frac{-x}{3x}=\lim\limits_{x\to-\infty}\frac{-1}{3}=-\frac{1}{3}$

3. (a) Since f has a vertical asymptote at $x = 2$, the limit on either side will be $+\infty$, or $-\infty$.
   We only need to check the signs:

   If $x\to2^-$, then $x<2$ and $(x-2)<0$. Therefore, $\lim\limits_{x\to2^-}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(+)}{(-)(+)}\infty=-\infty$.

   If $x\to2^+$, then $x>2$ and $(x-2)>0$. Therefore, $\lim\limits_{x\to2^+}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(+)}{(+)(+)}\infty=+\infty$.

   Hence, $\lim\limits_{x\to2}f(x)$ DNE.

   (b) If $x\to-3^-$, then $(x+3)<0$. Therefore, $\lim\limits_{x\to-3^-}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(-)}{(-)(+)}\infty=+\infty$.

   If $x\to-3^+$, then $(x+3)>0$. Therefore, $\lim\limits_{x\to-3^+}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(-)}{(-)(+)}\infty=+\infty$.

   Hence, $\lim\limits_{x\to-3}f(x)=+\infty$.

4. (a) $f'(x)=\frac{(30x)(x^2-100)-(15x^2)(2x)}{(x^2-100)^2}=\frac{-3000x}{(x^2-100)^2}$; f' is zero at $x = 0$ and undefined at $x = 10, -10$.

   (b) Increasing on $(-\infty,-10)$ and $(-10, 0)$;
   decreasing on $(0, 10)$ and $(10, +\infty)$.

$+$	-10	$+$	0	$-$	10	$-$

   (c) $\lim\limits_{x\to-10^-}\frac{15x^2}{x^2-100}=+\infty$; $\lim\limits_{x\to-10^+}\frac{15x^2}{x^2-100}=-\infty$; $\lim\limits_{x\to10^-}\frac{15x^2}{x^2-100}=-\infty$; $\lim\limits_{x\to10^+}\frac{15x^2}{x^2-100}=+\infty$

5. (a) $+\infty$; (b) $-\infty$; (c) DNE; (d) $+\infty$; (e) $+\infty$; (f) $+\infty$; (g) 1; (h) -2

6. (b) $y(2)$ must exist

   (d) $\lim\limits_{x\to2}y(x)$ must exist

   (f) $\lim\limits_{x\to2}y(x)$ must equal $y(2)$

Activity 4.3 – Continuity and L'Hôpital's Rule

1. (a) $\lim_{x\to 1}\frac{x^2-6x+5}{x^3-1}\overset{LR}{=}\lim_{x\to 1}\frac{2x-6}{3x^2}=-\frac{4}{3}$

(0/0)

(b) $\lim_{x\to -2}\frac{x^3+3x^2-4}{x^3+4x^2+4x}\overset{LR}{=}\lim_{x\to -2}\frac{3x^2+6x}{3x^2+8x+4}\overset{LR}{=}\lim_{x\to -2}\frac{6x+6}{6x+8}=\frac{3}{2}$

(0/0) (0/0)

(c) $\lim_{x\to 3}\frac{-2x^3+5x}{x-3}$ has the form –39/0, so L'Hôpital's Rule does not apply here. The limit is either +∞, –∞, or DNE (vert. asymp. at x = 3). We must check left- and right-hand limits: $\lim_{x\to 3^-}\frac{-2x^3+5x}{x-3}=\frac{(-)}{(-)}\infty=+\infty$; $\lim_{x\to 3^+}\frac{-2x^3+5x}{x-3}=\frac{(-)}{(+)}\infty=-\infty$; Therefore, $\lim_{x\to 3}\frac{-2x^3+5x}{x-3}$ DNE.

(d) $\lim_{x\to 0}\frac{3x^9+4x^5-x^3}{x^7-10x^2+4}=\frac{0}{4}=0$

(e) $\lim_{x\to 4}\frac{5x-20}{x^3-3x^2-4x}\overset{LR}{=}\lim_{x\to 4}\frac{5}{3x^2-6x-4}=\frac{1}{4}$

(0/0)

(f) $\lim_{x\to -1}\frac{x^3+3x^2+3x+1}{x^3+x^2-x-1}\overset{LR}{=}\lim_{x\to -1}\frac{3x^2+6x+3}{3x^2+2x-1}\overset{LR}{=}\lim_{x\to -1}\frac{6x+6}{6x+2}=\frac{0}{-4}=0$

(0/0) (0/0)

2. (a) $\lim_{x\to +\infty}\frac{11x^2-4x+7}{15x^2+2}\overset{LR}{=}\lim_{x\to +\infty}\frac{22x-4}{30x}\overset{LR}{=}\lim_{x\to +\infty}\frac{22}{30}=\frac{11}{15}$

(+∞/+∞) (+∞/+∞)

(c) (i) $\lim_{x\to +\infty}\frac{11x^{97}-4x^{33}+x^2-10}{15x^{97}+22x^{54}+6x^{12}-2}=\lim_{x\to +\infty}\frac{11x^{97}}{15x^{97}}=\frac{11}{15}$

(ii) $\lim_{x\to +\infty}\frac{4x^5+3x^3+x^2}{-x^5+8x-7}=\lim_{x\to +\infty}\frac{4x^5}{-x^5}=-4$

3. (a) (i) $\lim_{x\to -\infty}\frac{6x^2+4x}{5x^3-3x^2-2}\overset{LR}{=}\lim_{x\to -\infty}\frac{12x+4}{15x^2-6x}\overset{LR}{=}\lim_{x\to -\infty}\frac{12}{30x}=0$

(+∞/–∞) (–∞/+∞) (12/–∞)

(ii) $\lim_{x\to -\infty}\frac{9x^3-4x^2+3x-2}{x^2+3x}\overset{LR}{=}\lim_{x\to -\infty}\frac{27x^2-8x+3}{2x+3}\overset{LR}{=}\lim_{x\to -\infty}\frac{54x-8}{2}=-\infty$

(–∞/+∞) (+∞/–∞) (–∞/2)

(b) (i) $\lim_{x\to +\infty}\frac{6x^{50}-7x^9+3x^5-10x^2}{5x^{71}+21x^{33}-66x^{12}+1}=\lim_{x\to +\infty}\frac{6x^{50}}{5x^{71}}=\lim_{x\to +\infty}\frac{6}{5x^{21}}=0$

(ii) $\lim_{x\to -\infty}\frac{9x^{45}-10x^{17}+2x^{10}-10x^2+4}{x^{39}-21x^{30}+40x^{15}+x^8-9x}=\lim_{x\to -\infty}\frac{9x^{45}}{x^{39}}=\lim_{x\to -\infty}\frac{9x^6}{1}=+\infty$

4. Set $c(2)^2+7(2)=(2)^3-c(2)$ to get $4c+14=8-2c$. Therefore, $c=-1$.

Activity 5.1 – Exponential Growth and Decay

1. (a) If $M(t) = at + b$, then $M(0) = b = 500$, $M(2) = 2a + 500 = 245$, and $a = -127.5$.
   The model is $M(t) = -127.5t + 500$ mg, where t is hours after the injection.

   (b) If $M(t) = A(1-r)^t$, then $M(0) = A = 500$, $M(2) = 500(1-r)^2 = 245$, and $1 - r = 0.7$.
   The model is $M(t) = 500(0.7)^t$ mg, where t is hours after the injection.

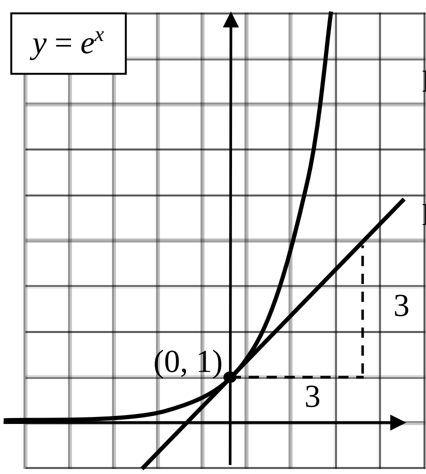
2. (a) $m(0) = 90$ grams
   (b) $m(40) \approx 27$ grams
   (c) decay rate of 0.03 = 3%

3. $P(t) = 77.2e^{0.016t}$ million tons, where t is years since 2004

4.

n compounding per yr	$\left(1+\frac{1}{n}\right)^n$ dollars after 1 yr
1 (yearly)	$\left(1+\frac{1}{1}\right)^1 = \$2.00000000$
12 (monthly)	$\left(1+\frac{1}{12}\right)^{12} = \$2.61303529$
365 (daily)	\$2.71456748
525,600 (every minute)	\$2.71827922
↓	↓
$+\infty$	\$2.718281828459…

5.



METHOD 1: $f'(0) = \frac{d}{dx}\left(e^x\right)\Big|_{x=0} = \frac{RISE}{RUN} = \frac{3}{3} = 1$

METHOD 2: $f'(0) = \frac{d}{dx}\left(e^x\right)\Big|_{x=0} = \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1$

Δx	–0.1	–0.01	–0.001	→	0	←	0.001	0.01	0.1
$\frac{e^{\Delta x}-1}{\Delta x}$	0.9516	0.9950	0.9995	→	1	←	1.0005	1.0050	1.0517

6. $\lim_{n\to+\infty}\left(1+\frac{r}{n}\right)^n = \lim_{mr\to+\infty}\left(1+\frac{r}{mr}\right)^{mr} = \lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^{mr} = \lim_{m\to+\infty}\left(\left(1+\frac{1}{m}\right)^m\right)^r = \left(\lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^m\right)^r$;

   The limit inside the outer parentheses is equivalent to $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e$, hence

$$\lim_{n\to+\infty}\left(1+\frac{r}{n}\right)^n = \left(\lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^m\right)^r = (e)^r = e^r.$$

Activity 5.2 – Derivative and Antiderivative of e^x

1. (a) $\frac{d}{dx}\left(5e^{4x}\right) = 5e^{4x} \cdot 4 = 20e^{4x}$

   (b) $\frac{d}{dt}\left(e^{2t-t^3}\right) = e^{2t-t^3} \cdot \left(2-3t^2\right)$

   (c) $\frac{d}{du}\left(e^u\left(1+2u^2-6u^4\right)\right) = \left(e^u\right)\left(1+2u^2-6u^4\right) + \left(e^u\right)\left(4u-24u^3\right) = e^u\left(1+4u+2u^2-24u^3-6u^4\right)$

   (d) $\frac{d}{dx}\left(\frac{x^2}{9e^{2x}+1}\right) = \frac{(2x)\left(9e^{2x}+1\right)-\left(x^2\right)\left(18e^{2x}\right)}{\left(9e^{2x}+1\right)^2}$

   (e) $\frac{d}{dx}\left(\frac{10e^x}{x^2+4e^{2x}}\right) = \frac{\left(10e^x\right)\left(x^2+4e^{2x}\right)-\left(10e^x\right)\left(2x+8e^{2x}\right)}{\left(x^2+4e^{2x}\right)^2}$

2. Set $f'(x) = (2x)\left(e^{6-x^2}\right) + \left(x^2\right)\left(e^{6-x^2} \cdot (-2x)\right) = e^{6-x^2}\left(2x-2x^3\right) = 0$ to get $x = -1, 0, 1$.

3. (a) $\int\left(5x-e^x\right)dx = \frac{5}{2}x^2 - e^x + C$

   (b) $\int 5e^{-2x}dx = -\frac{5}{2}e^{-2x} + C$

   (c) $\int_0^1 e^{-t}dt = \left(-e^{-t}\right)\Big|_0^1 = \left(-e^{-1}\right) - \left(-e^0\right) = -\frac{1}{e}+1$

4. (a) $+\infty$; (b) 0; (c) 0; (d) $+\infty$; (e) 0; (f) –5;

   (g) $\lim_{x\to+\infty}\frac{10e^{2x}}{x+4e^{2x}} \overset{LR}{=} \lim_{x\to+\infty}\frac{20e^{2x}}{1+8e^{2x}} \overset{LR}{=} \lim_{x\to+\infty}\frac{40e^{2x}}{16e^{2x}} = \frac{5}{2}$

5. Set $D'(t) = (-2)\left(e^{-5t}\right) + (-2t)\left(-5e^{-5t}\right) = e^{-5t}(-2+10t) = 0$ to get $t = 1/5$ s.

6. (c) $U(t) = 0.1496(1.1289)^t$ million users, where t is months since 12/31/2003

   (d) $U(48) = 50.5$ million users

   (e) $U'(t) = 0.1496 \cdot e^{0.1213t} \cdot 0.1213 = 0.0181 \cdot e^{0.1213t}$ million users per month, where t is months since 12/31/2003

   (f) $U'(48) = 6.11$ million users per month

Activity 5.3 – Implicit Differentiation and Inverse Functions

1. (a) $y' = -\frac{x}{y}$

   (b) $y' = -\frac{3y}{2x}$

   (c) $y' = \frac{-3x^2 - y^2}{2xy - 4}$ or $\frac{3x^2 + y^2}{4 - 2xy}$

   (d) $y' = -\frac{\sqrt{y}}{\sqrt{x}}$

   (e) $y' = \frac{5 - ye^{xy}}{xe^{xy}}$

2. $y' = \frac{2 - 2x}{2y - 8}$;

   Horizontal tangents: Set $2 - 2x = 0$ to get $x = 1$. Substitute $x = 1$ into the original equation to get a quadratic equation in y with solutions $y = 0$ and $y = 8$. The points at which the circle has horizontal tangents are (1, 0) and (1, 8).
   Vertical tangents: Set $2y - 8 = 0$ to get $y = 4$. Substitute $y = 4$ into the original equation to get a quadratic equation in x with solutions $x = -3$ and $x = 5$. The points at which the circle has vertical tangents are (–3, 4) and (5, 4).

3. (a) $f^{-1}(x) = \frac{x+1}{6}$

   (b) $g^{-1}(x) = x^2 - 9$

   (c) $h^{-1}(x) = \frac{4x}{x-2}$

4. Since $f'(x) = 2x$, $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2(f^{-1}(x))} = \frac{1}{2\left(\sqrt{x}\right)}$.

Activity 5.4 – Logarithmic Functions

1. (a) $x = \frac{1}{2}\ln(10)$

   (b) $x = e^5 - 3$

2. (a) $\ln\left(\frac{(x+2)\cdot\sqrt{x}}{\sqrt[3]{x^2+4}}\right) = \ln(x+2) + \frac{1}{2}\ln x - \frac{1}{3}\left(x^2+4\right)$

   (b) $\log_{10}(x+1)^2 = \log_{10}(4x)$, so $(x + 1)^2 = 4x$. Solve this quadratic equation to get $x = 1$.

3. (a) Set $36.5b^7 = 351.8$ to get $b = \left(\frac{351.8}{36.5}\right)^{1/7} \approx 1.3822$

   (b) Set $36.5(1.3822)^t = 73$ to get $t = \frac{\ln(2)}{\ln(1.3822)} \approx 2.14$ years

4. Set $400e^{1590k} = \frac{1}{2}\cdot 400$ to get $k = \frac{\ln\left(\frac{1}{2}\right)}{1590}$. Therefore, $A(t) = 400e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{1590}\right)t}$, and so

   $A(2000) = 400e^{\left(\frac{\ln\left(\frac{1}{2}\right)}{1590}\right)(2000)} \approx 167$ mg.

5. (a) $+\infty$

   (b) $-\infty$

   (c) 0

   (d) 0

6. (a) If $T(60) = 75 = 55 + (80-55)e^{-k(60)}$, then $25e^{-60k} = 20$, and $k = -\frac{1}{60}\ln\left(\frac{20}{25}\right) \approx 0.003719$.

   (b) Since $55 + (80-55)e^{-0.003719t} = 98.6$, we have $t = -\frac{1}{0.003719}\ln\left(\frac{43.6}{25}\right) \approx -149.55$. This is 149.55 minutes *before* 11:00 p.m. Therefore, the time of death was around 8:30 p.m.

Activity 5.5 – Derivatives and Antiderivatives of Exponentials and Logarithms

1. (a) $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$; (b) $\frac{d}{dx}(\log_b(x)) = \frac{1}{(\ln b)x}$; (c) $\frac{d}{dx}(\ln(2x-10)) = \frac{2}{2x-10} = \frac{1}{x-5}$

   (d) $\frac{d}{dx}\left(\log_{10}\left(3x^2+9x+7\right)\right) = \frac{6x+9}{(\ln 10)\left(3x^2+9x+7\right)}$; (e) $\frac{d}{dt}\left((\ln(2t+1))^3\right) = 3(\ln(2t+1))^2 \cdot \frac{2}{2t+1}$

   (f) $\frac{d}{du}\left(4u^2 \ln(u)\right) = (8u)(\ln(u)) + \left(4u^2\right)\left(\frac{1}{u}\right)$

2. (a) $y' = e^x$; (b) $y' = (\ln b) \cdot b^x$; (c) $f'(x) = (\ln 5) \cdot 5^x$

   (d) $A'(t) = 1000 \cdot \ln(1.03) \cdot (1.03)^{12t} \cdot 12$; (e) $g'(x) = \frac{\left((\ln 10) \cdot 10^x\right)(3x-1) - \left(10^x\right)(3)}{(3x-1)^2}$

3. (a) $\int\left(5^x - e^{5x}\right)dx = \frac{5^x}{\ln 5} - \frac{1}{5}e^{5x} + C$

   (b) $\int \frac{1-3x+4x^2}{x^2}dx = \int\left(\frac{1}{x^2} - \frac{3}{x} + 4\right)dx = -\frac{1}{x} - 3\ln|x| + 4x + C$

4. (a) $\lim_{x\to+\infty} \frac{\ln(7x-8)}{\ln(4x+2)} \overset{LR}{=} \lim_{x\to+\infty} \frac{\left(\frac{7}{7x-8}\right)}{\left(\frac{4}{4x+2}\right)} = \lim_{x\to+\infty} \frac{7}{7x-8} \cdot \frac{4x+2}{4} = \lim_{x\to+\infty} \frac{28x+14}{28x-32} = \frac{28}{28} = 1$

   (b) $\lim_{x\to 3} \frac{\ln(x^2-8)}{x-3} \overset{LR}{=} \lim_{x\to 3} \frac{\left(\frac{2x}{x^2-8}\right)}{1} = 6$

5. (a) $k(x) = 2^2$ is a constant function. Its derivative is $k'(x) = 0$.

   (b) $g(x) = x^2$ is a power function. Its derivative is $g'(x) = 2x$.

   (c) $h(x) = 2^x$ is an exponential function. Its derivative is $h'(x) = \ln 2 \cdot 2^x$.

6. (a) If $y = x^x$, then $\ln y = \ln x^x = x \ln x$, and $\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln x)$. Therefore, $\frac{y'}{y} = \ln x + 1$

   ,

   and $y' = y(\ln x + 1) = x^x(\ln x + 1)$.

   (b) If $y = x^n$, then $\ln y = \ln x^n = n \ln x$, and $\frac{d}{dx}(\ln y) = \frac{d}{dx}(n \ln x)$. Therefore, $\frac{y'}{y} = \frac{n}{x}$,

   and $y' = y\left(\frac{n}{x}\right) = x^n\left(\frac{n}{x}\right) = nx^{n-1}$.

   (c) $\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(3\ln x + \frac{1}{2}\ln(4x-11) - 5\ln\left(1+x^2\right)\right)$ yields $y' = \frac{x^3\sqrt{4x-11}}{(1+x^2)^5}\left(\frac{3}{x} + \frac{2}{4x-11} - \frac{10x}{1+x^2}\right)$.

7. $\Delta H = \int_{T_1}^{T_2} C_P(T)dT = \int_{290}^{1000}\left(a + bT + \frac{c}{T^2}\right)dT$

   $= \left(46.90T + \frac{31.51\times10^{-3}}{2}T^2 - \frac{-10.08\times10^5}{T}\right)\Big|_{290}^{1000} = 45261.142 \text{ J/mol}$

   $\Delta S = \int_{T_1}^{T_2} \frac{C_P(T)}{T}dT = \int_{290}^{1000}\left(\frac{a}{T} + b + \frac{c}{T^3}\right)dT$

   $= \left(46.90\ln|T| + 31.51\times10^{-3}T - \frac{-10.08\times10^5}{2T^2}\right)\Big|_{290}^{1000} = 74.940 \text{ J/mol-K}$

Activity 5.6 – Definite Integrals of Exponentials and Logarithms

1. (a) $\Delta x = 0.4$

   (b) $\int_0^2 3e^{x^2}\,dx \approx \left(3e^{(0.2)^2} + 3e^{(0.6)^2} + 3e^{(1)^2} + 3e^{(1.4)^2} + 3e^{(1.8)^2}\right)\cdot 0.4 \approx 45.390565$

2. (a) $\int_{-1}^{1} e^{5.2x}\,dx = \left(\dfrac{e^{5.2x}}{5.2}\right)\Bigg|_{-1}^{1} = \dfrac{e^{5.2}}{5.2} - \dfrac{e^{-5.2}}{5.2} \approx 34.86$

   (b) $\int_0^1 10^x\,dx = \left(\dfrac{10^x}{\ln 10}\right)\Bigg|_0^1 = \dfrac{10}{\ln 10} - \dfrac{1}{\ln 10} \approx 3.91$

3. (a) The initial rate is $a = 0.5$, so $R(t) = 0.5b^t$. Also, $R(120) = 0.5b^{120} = 0.25$. From this last equation, $b^{120} = 0.5$, so $b = \sqrt[120]{0.5} \approx 0.9942$. The formula for $R(t)$ is $R(t) = 0.5(0.9942)^t$ million gallons per minute.

   (b) $R(280) = 0.5(0.9942)^{280} \approx 0.0981$ million gallons per minute.

   (c) $\int_0^{280} 0.5(0.9942)^t\,dt = \left(\dfrac{0.5(0.9942)^t}{\ln(0.9942)}\right)\Bigg|_0^{280} \approx 69.0938$ million gallons

4. (a) $\int_1^x \dfrac{1}{t}\,dt = \left(\ln|t|\right)\Big|_1^x = \ln|x| - \ln|1| = \ln x$

   (b) $\int_1^x \frac{1}{t}\,dt = \ln x = 1$ implies that $x = e$.

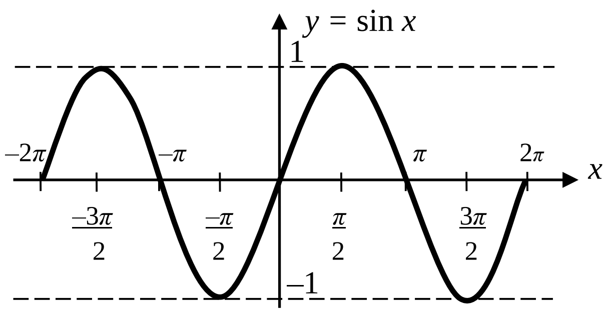
Activity 6.1 – The Cosine and Sine Functions

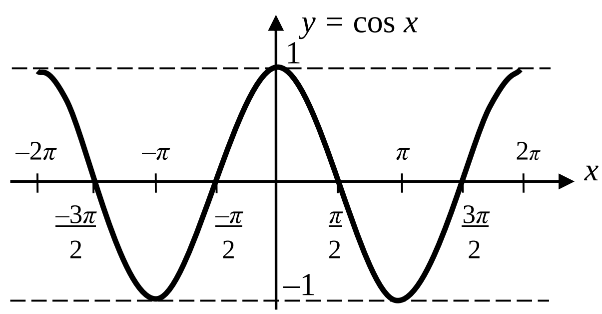
1.

θ	$x = \cos\theta$	$y = \sin\theta$
$0 = 0^\circ$	1	0
$\frac{\pi}{6} = 30^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3} = 60^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2} = 90^\circ$	0	1

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$0 = 0^\circ$	1	0	0
$\frac{\pi}{6} = 30^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3} = 60^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2} = 90^\circ$	0	1	undefined
$\pi = 180^\circ$	-1	0	0
$\frac{3\pi}{2} = 270^\circ$	0	-1	undefined
$\frac{5\pi}{6} = 150^\circ$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$
$\frac{4\pi}{3} = 240^\circ$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$

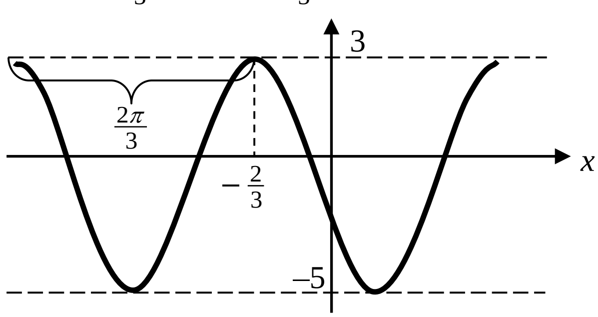
2.





3. (a) 4; (b) $\frac{2\pi}{3}$; (c) $-\frac{2}{3}$; (d) $y = -1$;

(e)



4. (a) $A = (3.96–1.60)/2 = 1.18$; $B = 2\pi/11$; $C = 0$; $D = (3.96+1.60)/2 = 2.78$; the model is $h(t) = 1.18\cos\left(\frac{2\pi}{11}t\right) + 2.78$ meters, where t is hours after 9:00 a.m. on July 22.

(b) $h(18) \approx 2.01$ meters; $h(47) \approx 2.61$ meters

(c) $C/B = –3$, so $C = –6\pi/11$; the model is $H(t) = 1.18\cos\left(\frac{2\pi}{11}t + \frac{6\pi}{11}\right) + 2.78$ meters, where t is hours after noon on July 22.

Activity 6.2 – Derivatives and Antiderivatives of Cosine and Sine

1. (a) $\frac{d}{dx}(\sin x) = \cos x$

   (b) $\frac{d}{dx}(\cos x) = -\sin x$

   (c) $\frac{d}{dx}(\sin 10x) = \cos 10x \cdot 10 = 10\cos 10x$

   (d) $\frac{d}{dx}\left(\frac{1}{2}\cos \pi x\right) = -\frac{1}{2}\sin \pi x \cdot \pi = -\frac{\pi}{2}\sin \pi x$

   (e) $\frac{d}{dx}\left(\sin x^3\right) = \cos x^3 \cdot 3x^2 = 3x^2 \cos x^3$

   (f) $\frac{d}{dx}\left(\cos^3 x\right) = 3\cos^2 x \cdot (-\sin x) = -3\sin x \cos^2 x$

   (g) $\frac{d}{dt}\left(t\sin\left(5t^2\right)\right) = (1)\left(\sin\left(5t^2\right)\right) + (t)\left(\cos\left(5t^2\right)\cdot 10t\right) = \sin\left(5t^2\right) + 10t^2\cos\left(5t^2\right)$

   (h) $\frac{d}{d\theta}\left(\frac{e^{5\theta}}{\sin(2\theta)}\right) = \frac{\left(5e^{5\theta}\right)(\sin(2\theta)) - \left(e^{5\theta}\right)(2\cos(2\theta))}{\sin^2(2\theta)}$

   (i) $\frac{d}{du}(\ln(\cos(3u))) = \frac{1}{\cos(3u)} \cdot (-3\sin(3u)) = \frac{-3\sin(3u)}{\cos(3u)}$

2. (a) $\int \cos x \, dx = \sin x + C$

   (b) $\int \sin x \, dx = -\cos x + C$

   (c) $\int 2\cos(3t)\, dt = \frac{2}{3}\sin(3t) + C$

   (d) $\int 0.5\cos\left(\frac{x}{4}\right)dx = 2\sin\left(\frac{x}{4}\right) + C$

   (e) $\int_0^2 \sin\left(\frac{\pi}{2}x\right)dx = \left(-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right)\Big|_0^2 = \frac{4}{\pi}$

3. (a) $\lim_{x\to 0}\frac{\sin(5x)}{\sin(8x)} \overset{LR}{=} \lim_{x\to 0}\frac{5\cos(5x)}{8\cos(8x)} = \frac{5}{8}$

   (b) $\lim_{x\to 2}\frac{\sin(2x-4)}{\cos(\pi x) - \frac{x}{2}} \overset{LR}{=} \lim_{x\to 2}\frac{2\cos(2x-4)}{-\pi\sin(\pi x) - \frac{1}{2}} = -4$

4. (a) $h(23) = 1.18\cos\left(\frac{2\pi}{11}(23)\right) + 2.78 \approx 3.77$ meters

   (b) $h'(t) = (1.18)\cdot\left(\frac{2\pi}{11}\right)\cdot\left(-\sin\left(\frac{2\pi}{11}t\right)\right) = -\frac{2.36\pi}{11}\sin\left(\frac{2\pi}{11}t\right)$ meters per hour, where t is hours after 9:00 a.m. on July 22.

   (c) $h'(23) = -\frac{2.36\pi}{11}\sin\left(\frac{2\pi}{11}(23)\right) \approx -0.36$ meters per hour; negative rate means tide is falling.

5. $y' = \frac{3-\cos x}{4+\sin y}$

Activity 6.3 – Other Trigonometric Functions

1. (a) $\tan x = \frac{\sin x}{\cos x}$

   (b) $\sec x = \frac{1}{\cos x}$

   (c) $\cot x = \frac{\cos x}{\sin x}$

   (d) $\csc x = \frac{1}{\sin x}$

2. (a) $\frac{d}{dx}(\tan x) = \sec^2 x$

   (b) $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

   (c) $\frac{d}{dx}(\cot x) = -\csc^2 x$

   (d) $\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

3. (a) $\frac{d}{dx}\left(e^{\sec x}\right) = e^{\sec x} \cdot \sec x \cdot \tan x$

   (b) $\frac{d}{dt}(\tan(\ln t)) = \sec^2(\ln t) \cdot \frac{1}{t}$

   (c) $\frac{d}{d\theta}(\theta \cot(2\theta)) = (1)(\cot(2\theta)) + (\theta)\left(-\csc^2(2\theta) \cdot 2\right)$

   (d) $\frac{d}{dx}\left(\tan^2 x\right) = 2\tan x \cdot \sec^2 x$

4. (a) $\int \sec x \tan x \, dx = \sec x + C$

   (b) $\int \sec^2 x \, dx = \tan x + C$

   (c) $\int \csc^2 x \, dx = -\cot x + C$

   (d) $\int \csc x \cot x \, dx = -\csc x + C$

5. (a) $\int \sec^2(3x+2)\, d\theta = \frac{1}{3}\tan(3x+2) + C$

   (b) $\int \frac{x^2 \sin x - x}{x^2}\, dx = \int \left(\sin x - \frac{1}{x}\right) dx = -\cos x - \ln|x| + C$

6. $\lim_{\theta \to 0} \frac{7\sin\theta}{4\tan\theta + \theta} \overset{LR}{=} \lim_{\theta \to 0} \frac{7\cos\theta}{4\sec^2\theta + 1} = \frac{7}{5}$

7. (a) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

   (b) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} = -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) = -\left(\frac{1}{\sin^2 x}\right) = -\csc^2 x$

   (c) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left((\cos x)^{-1}\right) = (-1)(\cos x)^{-2} \cdot (-\sin x) = \frac{\sin x}{\cos^2 x} = \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) = \sec x \tan x$

   (d) $\frac{d}{dx}(\csc x) = \frac{d}{dx}\left((\sin x)^{-1}\right) = (-1)(\sin x)^{-2} \cdot (\cos x) = -\frac{\cos x}{\sin^2 x} = -\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) = -\csc x \cot x$

Activity 6.4 – Inverse Trigonometric Functions

1. (a) $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

2. (a) $\frac{d}{dx}\left(\sin^{-1}\left(e^x\right)\right) = \frac{1}{\sqrt{1-\left(e^x\right)^2}} \cdot e^x = \frac{e^x}{\sqrt{1-\left(e^x\right)^2}}$

   (b) $\frac{d}{dx}\left(x^2 \arccos\left(x^5\right)\right) = (2x)\left(\arccos\left(x^5\right)\right) + \left(x^2\right)\left(-\frac{1}{\sqrt{1-\left(x^5\right)^2}} \cdot 5x^4\right) = 2x \arccos\left(x^5\right) - \frac{5x^6}{\sqrt{1-x^{10}}}$

   (c) $\frac{d}{dx}\left(\frac{\tan^{-1}(x)}{\ln x}\right) = \frac{\left(\frac{1}{1+x^2}\right)(\ln x) - \left(\tan^{-1}(x)\right)\left(\frac{1}{x}\right)}{(\ln x)^2}$

3. (a) $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

   (b) $\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos x + C$

   (c) $\int \frac{1}{1+x^2} dx = \arctan x + C$

4. (a) $\int \frac{-1}{\sqrt{1-(3x)^2}} dx = \frac{1}{3}\arccos(3x) + C$

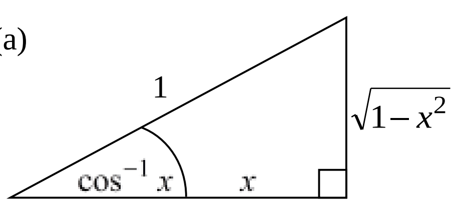
   (b) $\int_1^2 \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} dx = 2\arcsin\left(\frac{x}{2}\right)\Big|_1^2 = 2\arcsin(1) - 2\arcsin\left(\frac{1}{2}\right) = 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{\pi}{6} = \frac{2\pi}{3}$

   (c) $\int \frac{1}{16+x^2} dx = \frac{1}{16}\int \frac{1}{1+\left(\frac{x}{4}\right)^2} dx = \frac{1}{4}\arctan\left(\frac{x}{4}\right) + C$

5. (a) $\lim_{x \to +\infty} \arctan x = \frac{\pi}{2}$

   (b) $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

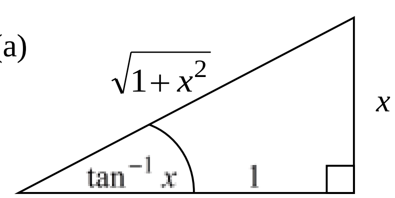
6. (a)



   (b) $\sin\left(\cos^{-1} x\right) = \sqrt{1-x^2}$

   (c) $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{1}{-\sin\left(\cos^{-1} x\right)} = -\frac{1}{\sqrt{1-x^2}}$

7. (a)



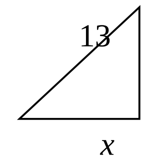
   (b) $\sec\left(\tan^{-1} x\right) = \sqrt{1+x^2}$.

   (c) $\sec^2\left(\tan^{-1} x\right) = 1 + x^2$

   (d) $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{\sec^2\left(\tan^{-1} x\right)} = \frac{1}{1+x^2}$

Activity 7.1 – Related Rates

1. **Related Variables Equation**: $x^2 + y^2 = 13^2$

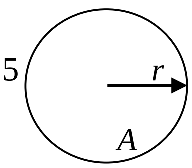


Given: $\frac{dx}{dt} = 1$; **Want**: $\frac{dy}{dt}$ when x = 12 and y = $\sqrt{169 - x^2} = \sqrt{169 - 144} = 5$

Related Rates Equation: $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0 \;\rightarrow\; 2(12)(1) + 2(5)\frac{dy}{dt} = 0 \;\rightarrow\; \frac{dy}{dt} = -2.4$

Answer: When the bottom of the ladder is 12 feet from the wall, the top of the ladder is falling at a rate of 2.4 ft/s.

2. **Related Variables Equation**: $A = \pi r^2$

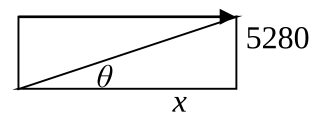


Given: $\frac{dA}{dt} = 4$; **Want**: $\frac{dy}{dt}$ when x = 12 and y = $\sqrt{169 - x^2} = \sqrt{169 - 144} = 5$

Related Rates Equation: $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \;\rightarrow\; 4 = 2\pi\sqrt{\frac{10}{\pi}} \cdot \frac{dr}{dt} \;\rightarrow\; \frac{dr}{dt} \approx 0.36$

Answer: When the area 10 mi^2, the radius of the spill is increasing at a rate of 0.36 mi/h.

3. **Related Variables Equation**: $\cot\theta = \frac{x}{5280}$

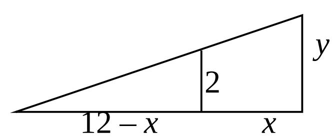


Given: $\frac{dx}{dt} = 400$; **Want**: $\frac{d\theta}{dt}$ when $\theta = 30° = \frac{\pi}{6}$

Related Rates Equation: $-\csc^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{5280} \cdot \frac{dx}{dt} \;\rightarrow\; -\frac{1}{\sin^2\left(\frac{\pi}{6}\right)} \cdot \frac{d\theta}{dt} = \frac{1}{5280} \cdot 400 \;\rightarrow\; \frac{d\theta}{dt} \approx -0.02$

Answer: When the angle of elevation θ is 30°, θ is decreasing at a rate of 0.02 rad/s.

4. **Related Variables Equation**: $\frac{12-x}{2} = \frac{12}{y}$ or $6 - \frac{x}{2} = \frac{12}{y}$

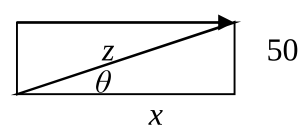


Given: $\frac{dx}{dt} = -1.2$; **Want**: $\frac{dy}{dt}$ when x = 6 and $y = \frac{24}{12-6} = 4$

Related Rates Equation: $-\frac{1}{2} \cdot \frac{dx}{dt} = -\frac{12}{y^2} \cdot \frac{dy}{dt} \;\rightarrow\; -\frac{1}{2} \cdot (-1.2) = -\frac{12}{(4)^2} \cdot \frac{dy}{dt} \;\rightarrow\; \frac{dy}{dt} = -0.8$

Answer: When the woman is 6 meters from the building, her shadow is decreasing at a rate of –0.8 m/s.

5. **Related Variables Equation**: $x^2 + 50^2 = z^2$



Given: $\frac{dx}{dt} = 2$; **Want**: $\frac{dz}{dt}$ when $\theta = \frac{\pi}{6}$ and $\frac{x}{z} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Related Rates Equation: $2x \cdot \frac{dx}{dt} = 2z \cdot \frac{dz}{dt} \;\rightarrow\; \frac{dz}{dt} = \frac{x}{z} \cdot \frac{dx}{dt}\; \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3} \approx 1.73$

Answer: When the angle of elevation is $\pi/6$ radians, the string must be let out at a rate of 1.73 ft/s.

6. **Related Variables Equation**: $P \cdot V^{1.2} = C$

Given: $\frac{dP}{dt} = -10$; **Want**: $\frac{dV}{dt}$ when V = 330 and P = 90

RRE: $\frac{dP}{dt} \cdot V^{1.2} + P \cdot \left(1.2V^{0.2}\right) \cdot \frac{dV}{dt} = 0 \;\rightarrow\; (-10) \cdot (330)^{1.2} + (90) \cdot \left(1.2(330)^{0.2}\right) \cdot \frac{dV}{dt} = 0$

Answer: At this instant, the volume is increasing by 30.56 cm^3/min.

Activity 7.2 – Graph Analysis Using First and Second Derivatives

1. (a) $f'(x) = (8 - 2x)e^x$

   (i) $x = 4$; (ii) $\left(4, 2e^4\right)$; (iii) none; (iv) $(-\infty, 4)$; (v) $(4, \infty)$

   (b) $f''(x) = (6 - 2x)e^x$

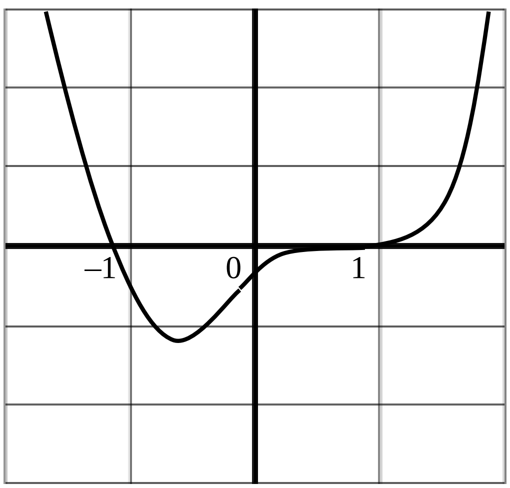
   (i) $\left(3, 4e^3\right)$; (ii) $(-\infty, 3)$; (iii) $(3, \infty)$

   (c) $f(x) = (10 - 2x)e^x$

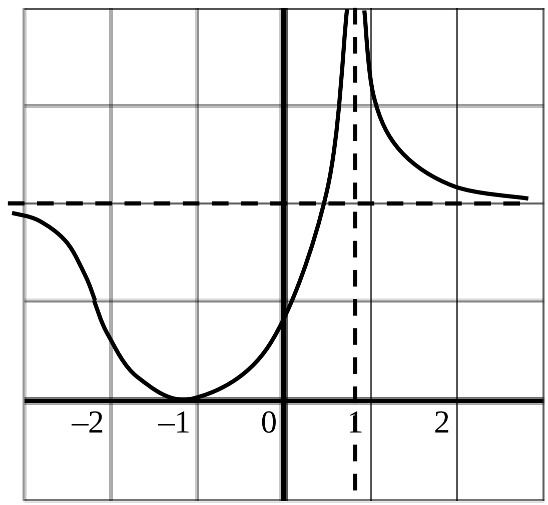
   (i) $x = 5$; (ii) $y = 10$

   (iii) $\lim\limits_{x\to+\infty} (10 - 2x)e^x = (-\infty)(+\infty) = -\infty$; $\lim\limits_{x\to-\infty} \frac{10-2x}{e^{-x}} \overset{LR}{=} \lim\limits_{x\to-\infty} \frac{-2}{-e^{-x}} = 0$

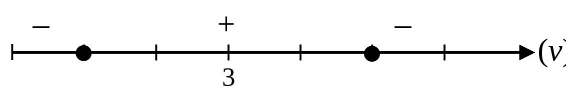
2.



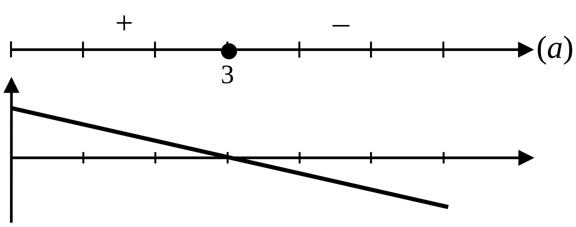
3.



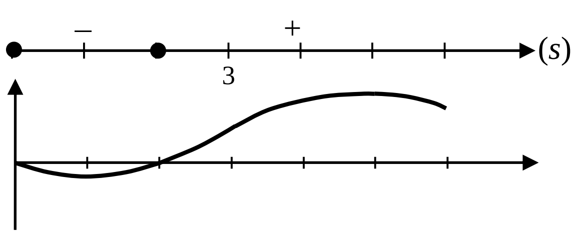
4. (a) Moving to left: (0, 1) ∪ (5, 6)
   Moving to right: (1, 5)
   At rest: $t = 1, 5$



   (b) Speeding up: (1, 3) ∪ (5, 6)
   Slowing down: (0, 1) ∪ (3, 5)



   (c) Left of origin: (0, 2)
   Right of origin: (2, 6)
   At origin: $t = 0, 2$



Activity 7.3 Solutions – Graph Analysis with the TI-84

1. $\text{Y1}(t) = 0.169t^3 - 1.571t^2 + 3.778t + 1.370$ million barrels per day, where t is years after 2008, $0 \le t \le 6$.

2. (a) t = 0.55 years after 2008 → in 2009
   t = 3.10 years after 2008 → in 2012
   t = 5.67 years after 2008 → in 2014

   (b) $\text{Y1}(1) = 3.75$ million barrels per day

   (c) $\text{Y1}'(1) = 1.14$; This is an increase of 1.14 million barrels per day per year.

   (d) At t = 1.63 (2010), the surplus was at a maximum of $\text{Y1} = 4.08$ million barrels per day.

   (e) At t = 4.59 (2013), the surplus was at a minimum of $\text{Y1} = 1.91$ million barrels per day.

3. $\text{Y1}'(t) = 0.506t^2 - 3.143t + 3.778$ million barrels per day per year, where t is years after 2008, $0 \le t \le 6$.

   (a) $\text{Y1}'(t) = 0$ when t = 1.63 and changes from positive to negative there. This verifies the local maximum at t = 1.63.

   $\text{Y1}'(t) = 0$ when t = 4.58 and changes from negative to positive there. This verifies the local minimum at t = 4.58.

   (b) The point of most rapid decline corresponds to the minimum of the slope graph, which occurs at t = 3.11. This corresponds to the year 2012. Algebraically,

   $$\text{Y1}''(t) = 1.012t - 3.143 = 0 \text{ when } t = 3.11.$$

   (c) $\text{Y1}'(3.11) = -1.10$; This is a decrease of 1.10 million barrels per day per year.

Activity 7.4 – The Extreme Value Theorem and Optimization

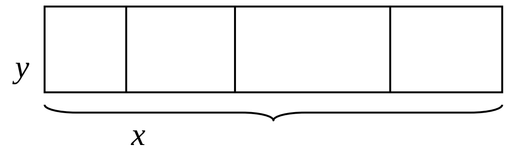
1. Length: $L = 2x + 5y$ (minimize)

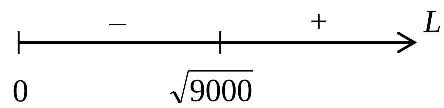
   Area: $A = xy = 3600 \rightarrow y = \frac{3600}{x}$

   $L(x) = 2x + 5\left(\frac{3600}{x}\right) = 2x + \frac{18000}{x}$

   $L'(x) = 2 - \frac{18000}{x^2} = 0 \rightarrow x^2 = 9000 \rightarrow x = \sqrt{9000}$

   Minimum length of fencing is $L\left(\sqrt{9000}\right) \approx 379$ ft.





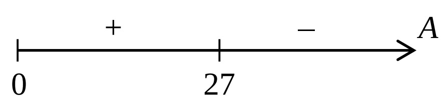
2. Area: $A = xy$ (maximize)

   Cost: $C = 10x + 30y = 540 \rightarrow y = \frac{540-10x}{30} = \frac{54-x}{3}$

   $A(x) = x\left(\frac{54-x}{3}\right) = 18x - \frac{1}{3}x^2$

   $A'(x) = 18 - \frac{2}{3}x = 0 \rightarrow x = 27$

   Maximum area is $A(27) = 243$ ft^2.



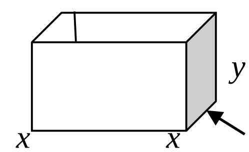
3. Volume: $V = x^2y$ (maximize)

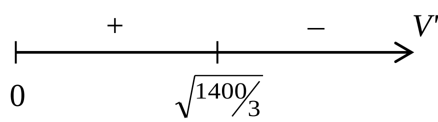
   Area: $A = x^2 + 4xy = 1400 \rightarrow y = \frac{1400-x^2}{4x}$

   $V(x) = x^2\left(\frac{1400-x^2}{4x}\right) = 350x - \frac{1}{4}x^3$

   $V'(x) = 350 - \frac{3}{4}x^2 = 0 \rightarrow x^2 = \frac{1400}{3} \rightarrow x = \sqrt{\frac{1400}{3}}$

   Maximum volume is $V\left(\sqrt{\frac{1400}{3}}\right) \approx 5040$ cm^3.





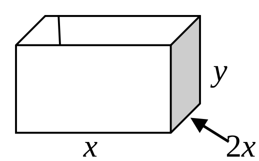
4. Cost: $C = 26x^2 + 30xy$ (minimize)

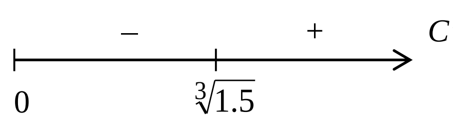
   Volume: $V = 2x^2y = 5.2 \rightarrow y = \frac{2.6}{x^2}$

   $C(x) = 26x^2 + 30x\left(\frac{2.6}{x^2}\right) = 26x^2 + \frac{78}{x}$

   $C'(x) = 52x - \frac{78}{x^2} = 0 \rightarrow x^3 = 1.5 \rightarrow x = \sqrt[3]{1.5}$

   Minimum cost is $C\left(\sqrt[3]{1.5}\right) = \$110.50$.





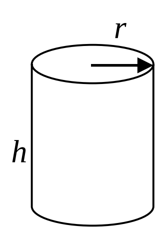
5. Volume: $V = \pi r^2 h$ (maximize)

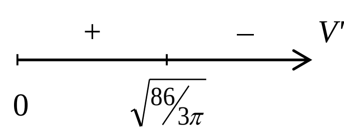
   Area: $A = \pi r^2 + 2\pi rh = 86 \rightarrow h = \frac{86-\pi r^2}{2\pi r}$

   $V(r) = \pi r^2\left(\frac{86-\pi r^2}{2\pi r}\right) = 43r - \frac{\pi}{2}r^3$

   $V'(r) = 43 - \frac{3\pi}{2}r^2 = 0 \rightarrow r^2 = \frac{86}{3\pi} \rightarrow r = \sqrt{\frac{86}{3\pi}}$

   Height is $h = \frac{86-\pi(86/(3\pi))}{2\pi\sqrt{86/(3\pi)}} \approx 3.02$ in. Maximum liquid is $V\left(\sqrt{\frac{86}{3\pi}}\right) \approx 86.6$ in^3.





Activity 7.5 – Differential Equations

1. (a) $y = C_0 e^{5t}$

   (b) $y = C_0 e^{-3t}$

   (c) $y = C_1 e^{\sqrt{25}t} + C_2 e^{-\sqrt{25}t} = C_1 e^{5t} + C_2 e^{-5t}$

   (d) $y = C_1 \cos\left(\sqrt{15}\, t\right) + C_2 \sin\left(\sqrt{15}\, t\right)$

2. $T(t) = 70 + 115 e^{-kt}$;

   $T(5) = 70 + 115 e^{-k(5)} = 170$ implies $k = -\frac{1}{5}\ln\left(\frac{100}{115}\right)$;

   $T(15) = 70 + 115 e^{-\left(-\frac{1}{5}\ln\left(\frac{100}{115}\right)\right)(15)} \approx 146\,°\text{F}.$

3. $y = \frac{5}{4} e^{4t} + \frac{7}{4} e^{-4t}$

4. $y(t) = -1.5\cos\left(\frac{1}{2}t\right) + 10\sin\left(\frac{1}{2}t\right)$ in

5. $y = C_0 e^{\pm kt} \quad \rightarrow \quad y' = \pm k\left(C_0 e^{\pm kt}\right) = \pm ky$

6. $y' = \sqrt{k}\left(C_1 e^{\sqrt{k}t}\right) - \sqrt{k}\left(C_2 e^{-\sqrt{k}t}\right)$

   $y'' = k\left(C_1 e^{\sqrt{k}t}\right) + k\left(C_2 e^{-\sqrt{k}t}\right) = k\left(C_1 e^{\sqrt{k}t} + C_2 e^{-\sqrt{k}t}\right) = ky$

7. $y' = \sqrt{k}\left(-C_1 \sin\left(\sqrt{k}\, t\right)\right) + \sqrt{k}\left(C_2 \cos\left(\sqrt{k}\, t\right)\right)$

   $y'' = k\left(-C_1 \cos\left(\sqrt{k}\, t\right)\right) + k\left(-C_2 \sin\left(\sqrt{k}\, t\right)\right) = -k\left(C_1 \cos\left(\sqrt{k}\, t\right) + C_2 \sin\left(\sqrt{k}\, t\right)\right) = -ky$

8. (a) $\int \frac{1}{y}\, dy = \int \pm k\, dt \quad \rightarrow \quad \ln|y| = \pm kt + C$

   (b) $\ln|y| = \pm kt + C \quad \rightarrow \quad e^{\ln|y|} = |y| = e^{\pm kt + C} \quad \rightarrow \quad y = \pm e^{C} e^{\pm kt} \quad \rightarrow \quad C_0 = \pm e^{C}$

Activity 8.1 – Sigma Notation and Summations

1. (a) 16; (b) 13

2. (a) $\sum_{k=1}^{n}\left(4k^3-2\right)=4\sum_{k=1}^{n}k^3-2\sum_{k=1}^{n}1=4\left(\frac{n^2(n+1)^2}{4}\right)-2(n)=n^2(n+1)^2-2n$

   (b) $\sum_{k=1}^{n}\left(1+4k+4k^2\right)=\sum_{k=1}^{n}1+4\sum_{k=1}^{n}k+4\sum_{k=1}^{n}k^2=n+4\left(\frac{n(n+1)}{2}\right)+4\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=n+2n(n+1)+\frac{2}{3}n(n+1)(2n+1)$

3. (a) $\sum_{k=1}^{63}\left(4k^3-2\right)=(63)^2(63+1)^2-2\cdot 63=16{,}256{,}898$

   (b) $\sum_{k=1}^{40}(1+2k)^2=40+2\cdot 40(40+1)+\frac{2}{3}\cdot 40(40+1)(2\cdot 40+1)=91{,}880$

4. (a) $\sum_{k=1}^{n}\left(2+\frac{1}{n}\cdot k\right)^2\left(\frac{1}{n}\right)=\sum_{k=1}^{n}\left(4+\frac{4}{n}\cdot k+\frac{1}{n^2}\cdot k^2\right)\left(\frac{1}{n}\right)$

   $=\sum_{k=1}^{n}\left(\frac{4}{n}\cdot 1+\frac{4}{n^2}\cdot k+\frac{1}{n^3}\cdot k^2\right)$

   $=\frac{4}{n}\sum_{k=1}^{n}1+\frac{4}{n^2}\sum_{k=1}^{n}k+\frac{1}{n^3}\sum_{k=1}^{n}k^2$

   $=\frac{4}{n}(n)+\frac{4}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=4+\frac{2(n+1)}{n}+\frac{(n+1)(2n+1)}{6n^2}$

   (b) $\sum_{k=1}^{n}3\left(-1+\frac{2}{n}\cdot k\right)^2\left(\frac{2}{n}\right)=\sum_{k=1}^{n}\left(\frac{6}{n}\cdot 1-\frac{24}{n^2}\cdot k+\frac{24}{n^3}\cdot k^2\right)$

   $=\frac{6}{n}(n)-\frac{24}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=6-\frac{12(n+1)}{n}+\frac{4(n+1)(2n+1)}{n^2}$

5. (a) $\sum_{k=1}^{3}\left(F(x_k)-F(x_{k-1})\right)=F(x_1)-F(x_0)+F(x_2)-F(x_1)+F(x_3)-F(x_2)$

   $=F(x_3)-F(x_0)$

   (b) $\sum_{k=1}^{4}\left(F(x_k)-F(x_{k-1})\right)=F(x_1)-F(x_0)+F(x_2)-F(x_1)+F(x_3)-F(x_2)+F(x_4)-F(x_3)$

   $=F(x_4)-F(x_0)$

   (c) $\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=F(x_n)-F(x_0)$

Activity 8.2 – The Definition of Net Area

1. $a = 1$; $b = 2$; $\Delta x = \frac{2-1}{n} = \frac{1}{n}$; $x_k^* = 1 + \frac{1}{n}\cdot k$; $f\left(x_k^*\right) = 3\left(1 + \frac{1}{n}\cdot k\right)^2 = 3 + \frac{6}{n}\cdot k + \frac{3}{n^2}\cdot k^2$;

$$f\left(x_k^*\right)\Delta x = \left(3 + \frac{6}{n}\cdot k + \frac{3}{n^2}\cdot k^2\right)\left(\frac{1}{n}\right) = \frac{3}{n} + \frac{6}{n^2}\cdot k + \frac{3}{n^3}\cdot k^2$$

$$\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \sum_{k=1}^{n}\left(\frac{3}{n} + \frac{6}{n^2}\cdot k + \frac{3}{n^3}\cdot k^2\right) = \frac{3}{n}\sum_{k=1}^{n} 1 + \frac{6}{n^2}\sum_{k=1}^{n} k + \frac{3}{n^3}\sum_{k=1}^{n} k^2$$

$$= \frac{3}{n}(n) + \frac{6}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 3 + \frac{3(n+1)}{n} + \frac{(n+1)(2n+1)}{2n^2}$$

$$\int_1^2 3x^2\,dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \lim_{n\to+\infty}\left(3 + \frac{3(n+1)}{n} + \frac{(n+1)(2n+1)}{2n^2}\right) = 3 + 3 + 1 = 7$$

2. $a = 0$; $b = 3$; $\Delta x = \frac{3-0}{n} = \frac{3}{n}$; $x_k^* = \frac{3}{n}\cdot k$; $f\left(x_k^*\right) = 2\left(\frac{3}{n}\cdot k\right)^2 - \left(\frac{3}{n}\cdot k\right) = \frac{18}{n^2}\cdot k^2 - \frac{3}{n}\cdot k$;

$$f\left(x_k^*\right)\Delta x = \left(\frac{18}{n^2}\cdot k^2 - \frac{3}{n}\cdot k\right)\left(\frac{3}{n}\right) = \frac{54}{n^3}\cdot k^2 - \frac{9}{n^2}\cdot k$$

$$\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \sum_{k=1}^{n}\left(\frac{54}{n^3}\cdot k^2 - \frac{9}{n^2}\cdot k\right) = \frac{54}{n^3}\sum_{k=1}^{n} k^2 - \frac{9}{n^2}\sum_{k=1}^{n} k = \frac{54}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{9(n+1)(2n+1)}{n^2} - \frac{9(n+1)}{2n}$$

$$\int_1^2 3x^2\,dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \lim_{n\to+\infty}\left(\frac{9(n+1)(2n+1)}{n^2} - \frac{9(n+1)}{2n}\right) = 18 - \frac{9}{2} = \frac{27}{2}$$

3. $\int_0^5 \left|t^2 - 2t\right|\,dt = -\int_0^2\left(t^2 - 2t\right)dt + \int_2^5\left(t^2 - 2t\right)dt = -\left(-\frac{4}{3}\right) + \left(\frac{54}{3}\right) = \frac{58}{3}$ m

4. (a) $\int_2^{-1} f(x)dx = -41$; (b) $\int_2^6 g(x)dx = -6$; (c) $\int_3^5 (4h(x) - 3)dx = 4\int_3^5 h(x)dx - \int_3^5 3dx = 18$

5. (a) $\int_a^a f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*)\cdot\frac{a-a}{n} = 0$

(b) $\int_b^a f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*)\cdot\frac{a-b}{n} = -\lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*)\cdot\frac{b-a}{n} = -\int_a^b f(x)dx$

(c) $\int_a^b k\cdot f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} k\cdot f(x_k^*)\cdot\frac{b-a}{n} = k\cdot\lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*)\cdot\frac{b-a}{n} = k\cdot\int_a^b f(x)dx$

(d) $\int_a^b (f(x) \pm g(x))dx = \lim_{n\to+\infty}\sum_{k=1}^{n}\left(f(x_k^*) \pm g(x_k^*)\right)\cdot\frac{b-a}{n} = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*)\cdot\frac{b-a}{n} \pm \lim_{n\to+\infty}\sum_{k=1}^{n} g(x_k^*)\cdot\frac{b-a}{n}$

$$= \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Activity 8.3 – Rolle's Theorem and the Mean Value Theorem

1. (a) Set $f'(c) = 3c^2 - 1 = \frac{f(2)-f(-2)}{2-(-2)} = \frac{8-(-4)}{4} = 3$ to get $c = \pm\sqrt{\frac{4}{3}}$ in the interval (–2, 2).

   (b) Set $g'(c) = -\frac{1}{c^2} = \frac{g(5)-g(3)}{5-3} = \frac{\frac{1}{5}-\frac{1}{3}}{2} = -\frac{1}{15}$ to get $c = \pm\sqrt{15}$, but only $\sqrt{15}$ is in (3, 5).

2. (a) Since f has a vertical asymptote at $x = 0$, it is not continuous on [0,4]. Since it is not continuous at $x = 0$, it is not differentiable at $x = 0$.

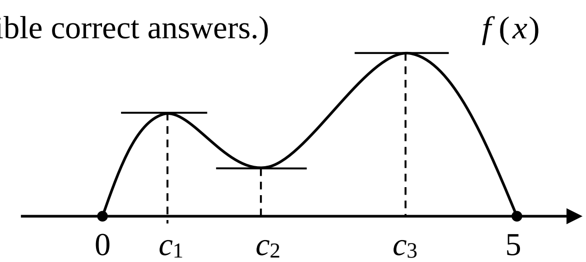
   (b) Since g only has one discontinuity at $x = 6$, it is continuous on [0, 4]. Since g' is undefined only at $x = 6$ (use the quotient rule), g is differentiable everywhere except at $x = 6$. In particular, it is differentiable on [0, 4].

   (c) Since h has a vertical asymptote at $x = –3$, it is not continuous on [–4 ,5]. Since it is not continuous at $x = –3$, it is not differentiable at $x = –3$.

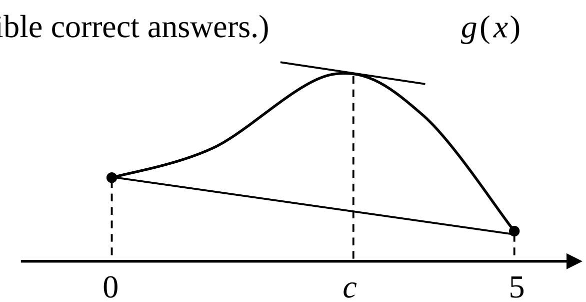
   (d) Since $F(x) = \begin{cases} -x^2 + x + 6, & \text{if } x < 3 \\ x^2 - x - 6, & \text{if } x \geq 3 \end{cases}$, it follows that $F'(x) = \begin{cases} -2x + 1, & \text{if } x < 3 \\ 2x - 1, & \text{if } x > 3 \end{cases}$.

   From the left of 3, F has a slope of –5, and from the right of 3, F has a slope of 5. Therefore, F is not differentiable at $x = 3$ (graph has a corner), and hence it is not differentiable on [1, 4]. It is continuous everywhere, however, as is seen by substituting $x = 3$ into the piecewise formulas for F.

3. (There are many possible correct answers.)



4. (There are many possible correct answers.)



5. Your average velocity between 8:00 and 8:03 is $\frac{\Delta\text{position}}{\Delta\text{time}} = \frac{4\text{ miles}}{3\text{ minutes}} \times \frac{60\text{ minutes}}{1\text{ hour}} = 80\text{ mi/h}$.
   By the MVT, there exists a time between 8:00 and 8:03 at which your instantaneous velocity was equal to your average velocity. In other words, you are guilty of traveling at a speed of 80 mi/h during this time interval.

Activity 8.4 – The Fundamental Theorem of Calculus (Part 1)

1. (a) $\int_1^{\sqrt{2}} \frac{8}{1+x^2}\,dx = 8\arctan(x)\Big|_1^{\sqrt{2}} = 8\arctan(\sqrt{2}) - 8\arctan(1) \approx 1.3593$

   (b) $\int_1^2 \frac{4}{x^3}\,dx = -\frac{2}{x^2}\Big|_1^2 = \left(-\frac{2}{4}\right) - \left(-\frac{2}{1}\right) = \frac{3}{2}$

   (c) $\int_0^2 \frac{4}{x^3}\,dx$ The FTC cannot be used since the integrand is not continuous at $x = 0$.

   (d) $\int_0^3 \frac{1}{x-2}\,dx$ The FTC cannot be used since the integrand is not continuous at $x = 2$.

   (e) $\int_4^9 6\sqrt{x}\,dx = \frac{6x^{3/2}}{3/2}\Big|_4^9 = 4(9)^{3/2} - 4(4)^{3/2} = 76$

   (f) $\int_0^{\pi} 3\cos x\,dx = 3\sin x\Big|_0^{\pi} = 3\sin\pi - 3\sin 0 = 0$

   (g) $\int_{-1}^1 e^{5x}\,dx = \frac{1}{5}e^{5x}\Big|_{-1}^1 = \frac{1}{5}e^5 - \frac{1}{5}e^{-5} \approx 29.681$

   (h) $\int_0^{0.5} \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x\Big|_0^{0.5} = \arcsin(0.5) - \arcsin(0) = \frac{\pi}{6}$

2. (a) NOTE THAT THE FOLLOWING COMPUTATION IS INCORRECT!

$$\int_{-1}^1 \frac{1}{x^2}\,dx = \int_{-1}^1 x^{-2}\,dx = -\frac{1}{x}\Big|_{-1}^1 = (-1) - (1) = -2$$

   (b) The graph is above the x-axis on [–1, 1], so the net area is positive, not negative.

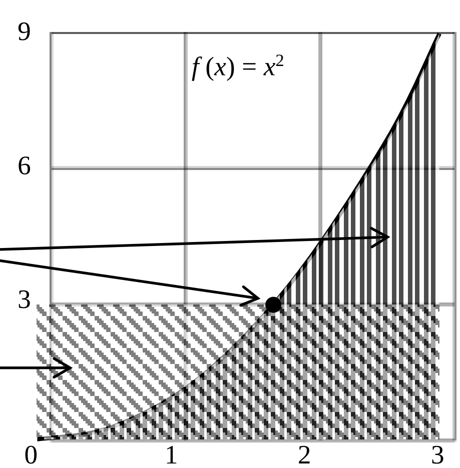
3. (a) Set $\int_0^3 x^2\,dx = (t^*)^2(3-0)$ so that $\frac{1}{3}x^3\Big|_0^3 = 3(t^*)^2$.

   Therefore, $9 = 3(t^*)^2$, and so $t^* = \sqrt{3} \approx 1.732$.

   (b) $(t^*, f(t^*))$

   (c) $\int_a^b f(x)\,dx$

   (d) $f(t^*)(b-a)$



Activity 8.5 – The Fundamental Theorem of Calculus (Part 2)

1. (a) $f'(x) = \sqrt{x^5 + 5x^2}$

   (b) $y(x) = -\int_1^x \frac{t^3}{e^t + 4} dt$, so $y'(x) = -\frac{x^3}{e^x + 4}$

   (c) $F'(x) = \arctan((2x)^2 + 10) \cdot 2 = 2\arctan(4x^2 + 10)$

   (d) $H'(x) = \frac{1}{\sqrt[3]{(e^x)^2 + 2e^x}} \cdot e^x = \frac{e^x}{\sqrt[3]{e^{2x} + 2e^x}}$

   (e) $g(x) = -\int_1^{5x^2} \cos(\ln(t))dt$, so $g'(x) = -\cos(\ln(5x^2)) \cdot 10x = -10x\cos(\ln(5x^2))$

2. (a) $G\left(\frac{1}{3}\right) = \int_1^1 e^{-t^2} dt = 0$

   (b) $G'(x) = e^{-(3x)^2} \cdot 3 = 3e^{-9x^2}$

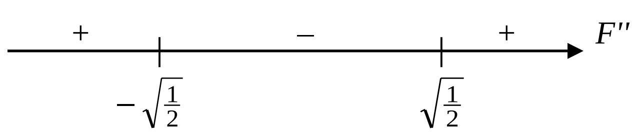
   (c) $G'(0) = 3e^{-9(0)^2} = 3$

3. (a) Set $F'(x) = (x^2 - 1)e^{x^2} \cdot 2x = 0$ to get $x = -1, 0, 1$. (Note that $e^{x^2} \neq 0$.)

   (b) Rewrite $F'(x) = (2x^3 - 2x)e^{x^2}$. Set

$$
\begin{aligned}
F''(x) &= (6x^2 - 2)e^{x^2} + (2x^3 - 2x)e^{x^2} \cdot 2x \\
&= 2(2x^4 + x^2 - 1)e^{x^2} \\
&= 2(2x^2 - 1)(x^2 + 1)e^{x^2} \\
&= 0
\end{aligned}
$$

   to get $2x^2 - 1 = 0$, or $x = \pm\sqrt{\frac{1}{2}}$ (Note that $e^{x^2} \neq 0$ and $x^2 + 1 \neq 0$.) A sign test shows that these are the locations of the inflection points:



Activity 8.6 – Integration by Substitution

1. (a) $\int e^{4x+5}dx = \int e^u \cdot \frac{1}{4}du = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + C = \frac{1}{4}e^{4x+5} + C$

   $u = 4x + 5$; $du = 4dx$; $dx = \frac{1}{4}du$

   (b) $\int x^2 e^{x^3}dx = \int e^u \cdot \frac{1}{3}du = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

   $u = x^3$; $du = 3x^2dx$; $x^2dx = \frac{1}{3}du$

   (c) $\int \frac{1}{9t+4}dt = \int \frac{1}{u}\cdot\frac{1}{9}du = \frac{1}{9}\int \frac{1}{u}du = \frac{1}{9}\ln|u| + C = \frac{1}{9}\ln|9t+4| + C$

   $u = 9t + 4$; $du = 9dt$; $dt = \frac{1}{9}du$

   (d) $\int \frac{x+6}{x^2+12x+10}dx = \int \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+12x+10| + C$

   $u = x^2 + 12x + 10$; $du = 2x + 12 = 2(x + 6)$; $(x + 6)dx = \frac{1}{2}du$

   (e) $\int \sin^5\theta\ \cos\theta\, d\theta = \int u^5 du = \frac{1}{6}u^6 + C = \frac{1}{6}\sin^6\theta + C$

   $u = \sin\theta$; $du = \cos\theta\, d\theta$

2. (a) $\int_0^1 x^3(1+2x^4)^3 dx = \int_1^3 u^3\cdot\frac{1}{8}du = \frac{1}{8}\int_1^3 u^3 du = \left.\frac{u^4}{32}\right|_1^3 = \frac{81}{32} - \frac{1}{32} = \frac{80}{32} = \frac{5}{2}$

   $u = 1 + 2x^4$; $du = 8x^3dx$; $x^3\,dx = \frac{1}{8}du$

   $x = 0 \to u = 1 + 2(0)^4 = 1$; $x = 1 \to u = 1 + 2(1)^4 = 3$

   (b) $\int_1^2 x\cdot\sqrt[3]{5-x^2}\,dx = -\frac{1}{2}\int_4^1 \sqrt[3]{u}\,du = \left.\left(-\frac{3}{8}u^{4/3}\right)\right|_4^1 = \left(-\frac{3}{8}\cdot 1^{4/3}\right) - \left(-\frac{3}{8}\cdot 4^{4/3}\right) = -\frac{3}{8} + \frac{3\sqrt[3]{4}}{2}$

   $u = 5 - x^2$; $du = -2xdx$; $xdx = -\frac{1}{2}du$

   $x = 1 \to u = 5 - (1)^2 = 4$; $x = 2 \to u = 5 - (2)^2 = 1$

   (c) $\int_0^{\pi/4} e^{\sin(2x)}\cos(2x)\,dx = \frac{1}{2}\int_0^1 e^u du = \left.\left(\frac{1}{2}e^u\right)\right|_0^1 = \left(\frac{1}{2}\cdot e^1\right) - \left(\frac{1}{2}\cdot e^0\right) = \frac{e}{2} - \frac{1}{2}$

   $u = \sin(2x)$; $du = 2\cos(2x)dx$; $\cos(2x)dx = \frac{1}{2}du$

   $x = 0 \to u = \sin(0) = 0$; $x = \pi/4 \to u = \sin(\pi/2) = 1$

3. (a) $\int \frac{3x^7}{(x^8+1)^2}dx = \frac{3}{8}\int\frac{1}{u^2}du = -\frac{3}{8u} + C = -\frac{3}{8x^8+8} + C$; $\int_0^1 \frac{3x^7}{(x^8+1)^2}dx = \left.\left(-\frac{3}{8x^8+8}\right)\right|_0^1 = \frac{3}{16}$

   $u = x^8 + 1$; $du = 8x^7dx$; $3x^7dx = \frac{3}{8}du$

   (b) $\int \frac{(\ln x)^2}{x}dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$; $\int_1^3 \frac{(\ln x)^2}{x}dx = \left.\frac{1}{3}(\ln x)^3\right|_1^3 = \frac{1}{3}(\ln 3)^3$

   $u = \ln x$; $du = \frac{1}{x}dx$

Activity 8.7 – Modeling Accumulated Change with the TI-84

1. $r(t) = -32t^2 + 933t + 507$ ft^3/min, where t is in minutes

2. t = 11.48 min

3. t = 14.58 min; $r(14.58)$ = 7308 ft^3/min

4. $\int_0^{10} \left(-32t^2 + 933t + 507\right)dt \approx 41053$ ft^3

5. $A(t) = \int \left(-32t^2 + 933t + 507\right)dt = -\frac{32}{3}t^3 + \frac{933}{2}t^2 + 507t + C$; since $A(0)$ = 5000, C = 5000.

   Therefore, $A(t) = -\frac{32}{3}t^3 + \frac{933}{2}t^2 + 507t + 5000$ ft^3, where t is minutes.

6. $A(20) = -\frac{32}{3}(20)^3 + \frac{933}{2}(20)^2 + 507(20) + 5000 \approx 116407$ ft^3

7. Set $-\frac{32}{3}t^3 + \frac{933}{2}t^2 + 507t + 5000 = 150000$ to get t = 27.55 min.